ASGER AABOE

SOME LUNAR AUXILIARY TABLES AND RELATED TEXTS FROM THE LATE BABYLONIAN PERIOD

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Synopsis

The present texts are concerned with a family of functions (Φ, F, G, Λ, X) from Babylonian lunar theory according to System A, all of them but X in evidence in the published corpus of texts, and each having the anomalistic month as its period. Rules for converting values of Φ into corresponding values of the other functions were under control, though they lacked motivation, but only the significance of F and G was known.

A text published by NEUGEBAUER in 1957 (the Saros text) and Text **E** below made it possible to identify all of these functions with reasonable certainty as well as to make astronomical sense of their established relations. Thus, for a given syzygy the associated values of the five functions have the following significance, beginning with the two that have long been identified:

daily progress of moon	=	F°
length of preceding month	=	$29^{d} + G^{H}$
length of subsequent 223 months	=	$6585^{d} + \Phi^{H}$
length of preceding 12 months	=	$354^{d} + \Lambda^{H}$
difference between a constant year and preceding 12 months	=	X^{d}.

All of these functions, save perhaps F, are artificial; they are first approximations, reflecting only the variation in lunar velocity, and resting on the preliminary assumption that syzygies are evenly distributed in longitude. G and, as I have discovered since this manuscript went to press, also Λ receive corrections for solar anomaly.

It appeared that when the values of Φ were to be used, the zig-zag function representing Φ was truncated at effective extrema (F was treated similarly).

Texts **A, B, C, D** below give evidence of aberrant Φ-G relations. Text **F** presents several variants of the function F, all truncated at the same values. Finally, a fragment joining the Saros text is published together with the relevant parts of the old text.

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Introduction

The present texts,[1] all in the British Museum, and all from unscientific excavations in Babylon, are concerned with Column Φ, in the terminology of ACT,[2] and related functions from Babylonian lunar theory.

Column Φ is a linear zig-zag function whose parameters are

$$M = 2{,}17,\ 4{,}48{,}53{,}20 \qquad p = \frac{1{,}44{,}7}{1{,}51{,}35} = 0;55{,}59{,}6,\ \ldots$$

$$m = 1{,}57{,}47{,}57{,}46{,}40 \qquad P = \frac{1{,}44{,}7}{7{,}28} = 13;56{,}39{,}6,\ \ldots$$

$$d = 2{,}45{,}55{,}33{,}20 \qquad \Pi = 7{,}28P = 1{,}44{,}7$$

where d is the difference corresponding to one synodic month. The period underlying Φ is the anomalistic month; indeed, Column Φ is exactly in phase with the unabbreviated Column F which represents daily lunar progress or, if one wishes, lunar velocity in degrees per day.

In a lunar ephemeris according to System A, Column Φ follows immediately upon the opening column listing year and month. It is a matter of experience, that Φ can be continued from one System A ephemeris to any other representing the same lunar phenomenon (either new or full moon); the corresponding families are called Φ_1 and Φ_2, respectively. Φ can thus be used for dating a text, and it is singularly useful for this purpose since it, when listed monthly, repeats itself exactly only after some 500 years (6247 months, to be precise). The dates I have affixed to the first four texts below

[1] A grant from the National Science Foundation enabled me to spend the summer of 1963 in London, studying cuneiform astronomical texts, among them the ones published here, in the British Museum; part of my subsequent work was supported by another NSF grant.

The texts are published through the courtesy of the Trustees of the British Museum.

I owe a particular debt of gratitude to Professor Abraham Sachs of Brown University: not only did he contribute directly to the present paper by giving me a transcription of Text **F**, but he patiently made himself available for checking difficult readings as well as for discussion of the issues as they arose.

[2] ACT = O. Neugebauer, *Astronomical Cuneiform Texts*, London, 1955.

are to be taken in this sense, i.e., they rest on the assumption that the Φ-values in these texts are connectible to the System A ephemerides in ACT.

Φ's only rôle in the fully developed System A scheme of the Seleucid period is to serve as the base for computing Column G, which denotes the excess over 29 days of a synodic month. G is a first approximation, taking into account only the variation in lunar velocity. It is convenient to describe G in terms of a linear zig-zag function Ĝ whose parameters are

$$M = 5;\ 4{,}57,\ 2{,}13{,}20^{H}$$

$$m = 2;\ 4{,}59{,}45{,}11,\ 6{,}40^{H}$$

$$d = 0;25{,}48{,}38{,}31,\ 6{,}40$$

where the units are large hours[3] and the difference corresponds to one synodic month.

Ĝ has the same period (the anomalistic month) as Φ and F, but is out of phase with them, so that the maximum of Ĝ occurs slightly after the minimum of Φ. G agrees with Ĝ on stretches of its ascending and descending branches, but is of a more complicated character and smoother appearance near its extrema which are

$$M = 4;56{,}35{,}33{,}20$$

$$m = 2;40$$

again in large hours.

The values of G near its extrema are derived from those of Φ according to a scheme which is given in tabular form in ACT, page 60. This scheme is the subject of several procedure texts (e.g., ACT No. 205, 206, 207, 207a, 207b), and it is followed in the System A ephemerides.

The texts **B, C, D** below do not agree with this scheme. The disagreement cannot be due to faulty interpolation, for in one case (Text **B,** Obverse, line 20) the Φ-value is precisely in the ACT interpolation scheme, and in another (Text **C,** line 7) the Φ-value should imply that G be equal to Ĝ which it is clearly not. Even the assumption that all three texts are consistent does not provide a sufficient base for reconstructing an underlying scheme.

These three texts are not the sole evidence for aberrant Φ-G relations. ACT No. 207ca is a procedure text which gives a variant of the usual scheme. Even though I have recently joined a fragment to this text[4], it seems unlikely that it can be brought in agreement with the present texts.

[3] 1 day = 6 large hours (H) = 6,0 (time) degrees. The large hour (= 1,0 time degrees) is a modern unit devised to avoid the use of Babylonian time degrees which often might necessitate comments.

[4] B. M. 40611 (81–4–28, 156) joins the upper edge of ACT No. 207ca. A photograph of the rejoined tablet may yield readings which are not yet possible. This text will be published together with some other additions to ACT.

The rules for converting Φ into G are presented without motivation, even in the ACT procedure texts, so all that Φ is used for in the ephemerides is essentially to locate a given moment within the anomalistic month, a task for which F might, incidentally, seem more naturally suited.[5] It is, therefore, very possible to compute a System A lunar table without knowing what Φ represents, and the precise meaning of Φ, and its original rôle, remained

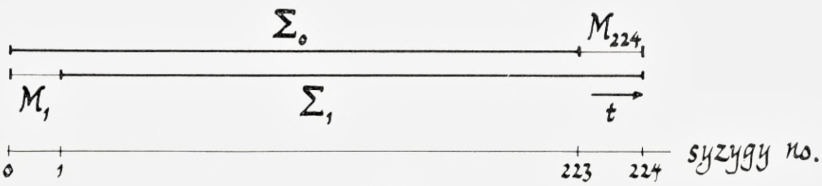

Fig. 1.

prominent among the unsolved problems in the ACT material. It was not until Neugebauer in 1957 published a difficult procedure text – I shall call it the Saros text[6] – that the discussion of this problem rose above mere guessing, as we now can see.

It appeared, entirely unexpectedly, that Φ has to do with the behaviour of the moon at 18-year intervals (more precisely, at intervals of 223 synodic months, the "Saros" period).[7] The crucial relation was, as Neugebauer rightly pointed out, that the monthly difference for Φ is the same as the difference in G-values 223 months apart, at least when Φ and G go from one linear branch to another of the same kind. This settled first that Φ measures time in large hours,[8] as does G. Second, when we disregard the restriction of Φ and G to branches of the same kind, this relation suggests that the difference between Φ and the length of 223 months (or one Saros) is constant. This may be seen by the following argument. Let Σ_0 and Σ_1 be the lengths of two Saroi, Σ_1 beginning one month later than Σ_0; let further M_1 be the month preceding Σ_1 and M_{224} that following Σ_0 (my choice of indices anticipates, that the length of a Saros is associated with its initial syzygy, and that of a month with its final syzygy). We then have (see Figure 1)

[5] There is, in fact, a procedure text (ACT No. 208) giving rules for transforming F into G.

[6] O. Neugebauer, *"Saros" and Lunar Velocity in Babylonian Astronomy.* Mat. Fys. Medd. Dan. Vid. Selsk. *31*, no. 4 (1957). I shall refer to it as the Saros paper.

[7] I shall here employ "Saros" to mean 223 synodic months. For a history of the use of "Saros" see O. Neugebauer, *The Exact Sciences in Antiquity.* 2nd edition. Providence: Brown University Press, 1957, p. 141 ff.

[8] Thus, e. g., $M_{\Phi} = 2;17,4,48,53,20^{\mathrm{H}}$.

$$
\begin{aligned}
\Sigma_1 - \Sigma_0 &= M_{224} - M_1 \\
&= (29^d + G_{224}) - (29^d + G_1) \\
&= G_{224} - G_1.
\end{aligned}
$$

Thus, if the relation

$$G_{224} - G_1 = \Phi_1 - \Phi_0$$

holds throughout, as suggested by the Saros text, then

$$\Sigma_1 - \Sigma_0 = \Phi_1 - \Phi_0$$

or

$$\Sigma - \Phi = \text{constant.}$$

It is, therefore, a reasonable hypothesis that even as G is the excess over 29^d of a synodic month, so Φ is the excess over a whole number of days of 223 synodic months, taking into account only the effect of a variable lunar velocity. In his recent book,[9] B. L. van der Waerden states this hypothesis and proceeds to show that it implies several of the established relations of Φ to F and G. He further mentions in passing, that in order to have agreement between Φ and the fine-structure of G one must assume that, whenever its values are to be used, Φ be truncated near its maximum and its minimum.

Text **E** relates Φ to Column Λ, which was already known from ACT No. 207d and 207e, though its significance was dark, and to the new Column X, as I call it. This text turns out to be the first in which the values of Φ are in active use, and van der Waerden's assumption is happily confirmed in that Φ appears truncated at the values 2;13,20 and 1;58,31,6,40. The structure of the text shows further that the difference in Φ over 12 months is the same as the difference in Λ for 223 months. Thus it follows that Λ indicates the length of 12 consecutive months. Its values are such that I believe it denotes the excess of a 12 month interval over a whole number of days (this can be negative). If this is so, it is possible to interpret Column X as the variable epact, that is, the difference (in days) between a year of constant length and the variable length of 12 months, ignoring all effects but that of lunar anomaly.

The technique displayed in Text **E** can be used, with obvious modifications, to construct a table for converting Φ into G; this I did and reached excellent, though not complete, agreement with the ACT scheme. The agreement is good enough, though, to make it perfectly clear that the rule suggested by the Saros text (the difference in G for 223 months is the same as the difference in Φ for one month) holds without any restriction, if Φ is truncated. This was

[9] B. L. van der Waerden, *Anfänge der Astronomie.* Grossingen, 1966, p. 148 ff.

the only assumption used in the argument above, so it can no longer be doubted that Φ measures the Saros and Λ the 12 month year, even as G measures the month.

Λ and G can thus be derived from Φ and an initial value for each, and the derivation makes astronomical, as well as arithmetical, sense. It still remains to be explained how the initial values were determined; I can, however, show that they are independently chosen, or at least not related in the obvious way.

Once it was realised that 2;13,20 and 1;58,31,6,40 are the effective extrema of Φ, several passages in previously known texts became significant, and I shall here draw attention to some of them.

ACT No. 200, Section 5, is concerned with the relations between F and Φ. It first gives the parameters for the standard, unabbreviated Column F:

$$M = 15;56,54,22,30^{0/d}$$

$$m = 11;\ 4,\ 4,41,15$$

$$d = 0;42$$

It then relates that to M_Φ corresponds M_F, and to m_Φ corresponds m_F. Thus, of course, one can determine the constants in the relation

$$F = c_1\Phi + c_2.$$

In fact, the text later gives $c_1 = 15,11,15$.

But after this we are told that to

$$\Phi = 2,13,20 \qquad \text{corresponds } F = 15$$

and to

$$\Phi = 1,58,31,6,40 \text{ corresponds } F = 11;15.$$

It used to appear unmotivated to single out these pairs of values (they are, of course, in agreement with the conversion rule), but now this passage suggests strongly that the effective extrema of F are 15 and 11;15, which, as we shall see, is so.

In three, I suspect rather old, lunar texts[10] we do indeed find Φ and F truncated at 2,13,20 and 15, respectively. These texts have, however, no opportunity to do similarly near the minima; this is not odd, for while it is impossible to avoid reaching values in excess of 2;13,20 and 15, when

[10] These lunar texts will be published shortly by A. Sachs and myself.

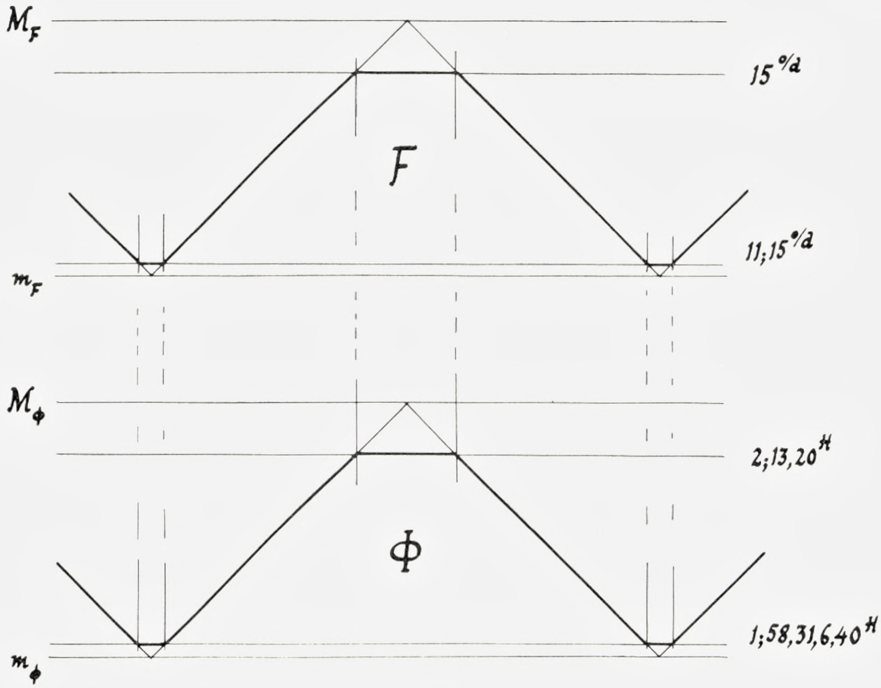

Fig. 2.

proceeding in monthly steps, 1;58,31,6,40 and 11;15 are so near the minima that it is certainly not always that these values would come into play.

In a table of effective Φ-values, 2,13,20 is thus very conspicuous; it is, therefore, not surprising that it appears that the number 2,13,20 is used in procedure texts as a proper name for Column Φ. Neugebauer lists in ACT I, page 212, several curious usages of 2,13,20, all of which make sense if for "2,13,20" we simply read "Column Φ".

In particular, ACT No. 204, Section 6, begins with the statement (Reverse, line 9):

2,13,20 *šá* u₄-1-kam *aná* u₄-14-kam *aná šu-ṣu-ú* 3[0 . . .]

Neglecting the final number, this may then be translated:

Column Φ for the 1st day to (Column Φ for) the 14th day for you to transform

or, more freely, using the terminology for Φ for new and full moon:

To transform Φ_1 into Φ_2.

The number following this sentence, broken off but for its beginning, must then be

$$20,39,48,53,20 = 15 \cdot d_{\Phi^*}$$

where

$$d_{\Phi^*} = 1{,}22{,}39{,}15{,}33{,}20$$

is the difference corresponding to 1 tithi[11] for Φ^*, the "daily" Φ (see ACT I, page 45).

As indicated, NEUGEBAUER reads 3[0 . . .], and PINCHES's hand copy (LBAT[12] No. 96) shows three corner wedges with a slightly larger space between the second and third than between the first and second. This agrees well with the restoration 20,39,48,53,20.

The passage can then be rendered thus, following NEUGEBAUER's restoration from line 10 and on:

9 *To transform* Φ_1 *into* Φ_2. 20,39,48,53,20
10 *add and subtract. That which exceeds* 2,17,4,48,53,20 *from* 2,17,4,48,
11 53,20 *subtract. That which goes below* 1,57,47,57,46,40 *to* 1,57,
12 47,57,46,40 *add and put it down . . .*

The rule works, of course. As an example, consider the following value of Φ_1 corresponding to S.E. 194, VII (ACT No. 13, Obverse, line 8)

	1,59,33,31, 6,40	
Augmented by:	20,39,48,53,20	
it is	2,20,13,20.	
This exceeds	2,17, 4,48,53,20	$(= M_\Phi)$
by	3, 8,31, 6,40	
which, subtracted from M_Φ, yields	2,13,56,17,46,40	

and this is precisely the value for Φ_2 corresponding to S.E. 194, VIII (ACT No. 13, Reverse, line 8).

Incidentally, the values 2,13,20 and 1,58,31,6,40 are assumed by Φ_2 on both an ascending and a descending branch. M_Φ is a value belonging to Φ_2, while m_Φ belongs to Φ_1. That the extrema are assumed implies symmetry in the sense that a value occurring on an ascending branch also occurs on a descending branch (thus it happens, that Φ_2 in Text **B** in part overlaps with ACT No. 1, but in reversed order).

Text **F** is a procedure text offering certain anomalies. It is concerned with several variants of Column F, all truncated at 15 and 11;15. One of the variants has, curiously enough, the same period as Column F in System B, while another probably has that of the standard Column F of System A.

[11] 1 synodic month = 30 tithis.

[12] LBAT = A. J. SACHS (ed.), *Late Babylonian Astronomical and Related Texts, copied by T. G. Pinches and J. N. Strassmaier.* Brown University Press, Providence, 1955.

Finally, I have joined yet a small fragment to Neugebauer's Saros text. This and the adjoining parts of the old text are published at the end of this paper, together with a discussion of a few of the old passages which can now be understood in the light of the above results about Φ.

Before presenting the texts themselves, I shall briefly argue, that the identification of Φ as the excess of 223 synodic months over a whole number of days implies that Φ is in phase with F.[13] The argument proceeds from the fact, that 239 anomalistic months are slightly longer than 223 synodic months. Since the progress, l_a, in longitude of the moon in one anomalistic month is constant, its progress, L_a, in 239 anomalistic months (a constant time T_a) is also constant. We now assume, that we are dealing with syzygies (either conjunctions or oppositions of the moon) distributed evenly in respect of *longitude*, and are then concerned with the variation in the corresponding time intervals induced by a variable lunar velocity. Let the constant progress of the moon from one syzygy to the next be l_s, and its progress in 223 synodic months be the constant amount L_s ($= 223\ l_s$).

Consider now a certain syzygy S_0 (see Figure 3). Associated with S_0 are values of Φ and F, say, Φ_0 and F_0. When the moon after 223 synodic months (of variable length) reaches the syzygy S_{223}, it will have travelled the distance L_s; let the corresponding time be T_s. We then have:

$$T_s = 6585^{\mathrm{d}} + \Phi_0{}^{\mathrm{H}}.$$

If from S_{223} we go L_a back in longitude, we shall reach a point which precedes S_0 by the amount l, where

$$l = L_a - L_s.$$

The constant l is small, but positive.
The corresponding time:

$$t = T_a - T_s$$

is variable, but small (and positive); we may, therefore, assume that the moon's velocity during t is F_0. We then have

$$t = \frac{l}{\mathrm{F}_0},$$

so

$$\frac{l}{\mathrm{F}_0} = T_a - T_s$$

$$= T_a - 6585^{\mathrm{d}} - \Phi_0.$$

[13] This argument is, in essence, one given me by Dr. John Britton in 1965, when he was still my student. It happens to be virtually identical with the one which later appeared in v. d. Waerden's book (cf. note 9).

Introducing the constant

$$c = T_a - 6585^{d},$$

i.e., the excess of 239 anomalistic months over a whole number of days, we get the relation between Φ and F:

$$\frac{l}{\mathrm{F}} = c - \Phi, \tag{1}$$

where l and c are constants.

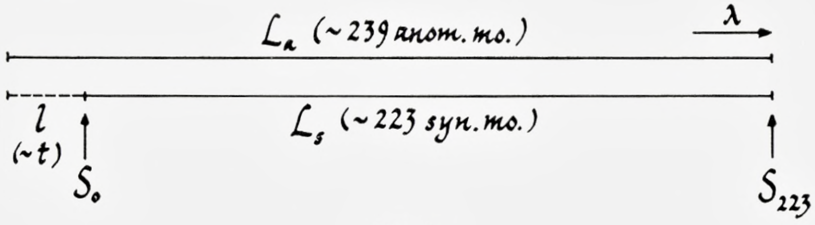

Fig. 3.

From (1) it follows immediately, that when Φ assumes its maximum then so does F, and analogously for their minima. Thus Φ and F are in phase.

If one chooses to represent the periodic functions Φ and F by zig-zag functions, the relation (1) cannot, of course, hold exactly.

To test whether the relation (1) may have been envisaged by the Babylonian astronomers, I introduced into it two pairs of Φ and F-values. It was here natural to pick the effective extrema, viz.

$$\Phi = 2;13,20^{\mathrm{H}} \qquad \mathrm{F} = 15^{0/\mathrm{d}}$$

and

$$\Phi = 1;58,31,6,40^{\mathrm{H}} \qquad \mathrm{F} = 11;15^{0/\mathrm{d}}.$$

These values yielded:

$$c = 2;57,46,40^{\mathrm{H}}$$

and

$$l = 1;51,\ 6,40^{0}$$

It is remarkable, that all these parameters, including c and l, are very simple regular numbers.

Further, the value for c implies a value for the anomalistic month, for c is the excess over 6585^{d} of 239 anomalistic months. This value is $27^{d};33,15,43,22,30, \ldots$ which compares rather well with the value given in the Saros text, viz., $27^{d};33,16,30$.

Though the agreement is not perfect, it still establishes beyond any doubt that the integral number of days is 6585, as I have tacitly assumed. Thus one would look in vain for a direct connexion between the mean values (or initial values) of Φ and G, for any reasonable mean value of G would imply a length of 223 months far in excess of 6585^d. It is well known that G is so adjusted that no correction to the month length is necessary when the solar progress is 30^0 per month, and that the correction (Column J) is negative on the arc of low solar velocity. Thus the mean value of G is high, and it is not strictly true that the length of the Saros, as used in the argument on page 6, is

$$\sum_{i=1}^{223} (29^d + G_i).$$

However, since the argument only turns on differences in Φ and in G, it is independent of the mean values, and so is still valid.

Texts A, B, C, D

Text A: B. M. 36994 (= 80-6-17, 738).
Contents: Φ_2 for S.E. 6, I–IX.
Transcription: Table 1.
Photograph: Plate I.

TABLE 1.

		Φ_2	$(\hat{G}_2)$
S.E. 6, I	1.	[2, 11, 2, 35]⌈33⌉[20]	3, 12, 51, 51, 6, 40
II		[2, 1]⌈3⌉, 48, 31, 6[40]	2, 47, 3, 12, 35, 33, 20
III		[2, 16, 34, 26, [40]	2, 21, 14, 34, 4, 26, 40
IV		[2, 14, 49, 15, 33, 20]	2, 14, 33, 34, 48, 53, 20
V	5.	2, 12, 3, 20	2, 40, 22, 13, 20
VI		2, 9, 17, 24, 2[6, 40]	3, 6, 10, 51, 51, 6, 40
VII		2, 6, 31, 28[53, 20]	3, 31, 59, 30, 22, 13, 20
VIII		2, 3, 45, 33, 20]	3, 57, 48, 8, 53, 20
IX		[2, 0]⌊59⌋[37, 46, 40]	4, 23, 36, 47, 24, 26, 40
X	10.	[1, 58, 13, 42, 13, 20]	4, 49, 25, 25, 55, 33, 20

Text A: BM 36994.

This is a flake with left and, perhaps, top edge preserved. There is no room for an initial date column, so the text is not a fragment of an ephemeris. In the transcription I have supplied the corresponding values of $\hat{G}_2$.

Text B: B. M. 36824 (= 80-6-17, 563) + 37222 (= 80-6-17, 976).
Contents: Obverse: Φ_2, G_2 for S.E. 35, I to S.E. 36, X.
Reverse: Φ_1, G_1 for S.E. 35, XI to S.E. 37, I.
Transcription: Table 2.
Photograph: Plate I.

TABLE 2.

Obv.

	Φ_2	G_2
S.E. 35, I	[2, 1,13]⌜4⌝[2, 13,20]	[4, 44,28, 8,53,20]
II	[2, 3]59, 37[46, 40]	[4, 18, 39,30,22, 13,20]
III	[2, 6]45, 33, 2[0]	[3, 52, 50,51, 51, 6,40]
IV	[2, 9],31,28,53,20	⌜3⌝[27, 2,13,20]
V	[2, 13],17, 24, 26,40	3, [1, 13, 34,48,53,20]
VI	[2, 15] 3, 20	2[//////////////////]
VII	[2, 16]20, 22, 13,20	2[//////////////////]
VIII	[2, 13, 3]4, 26, 40	2[//////////////////]
IX	[2, 10, 48, 3]1, 6,40	2[//////////////////]
X	[2, 8, 2, 3]⌜5⌝33,20	3, 17,49, 8, 8,[3,20]
XI	[2, 5, 16], 40	3, 43, 37, 46, 4[0]
XII	[2, 2,30]44,26,40	4, 9, 26, 25, 11, 6, 4[0]
36, I	[1, 59]44,48,53,20	4, 35, 15, 3, 42, 13,20
II	[1, 58, 37] 2, 13, 20	4, 56
III	[2, 1, 2]2, 57, 46, 40	4, 43, 1, 43, 42, 13, 20
IV	[2, 4] 8, 53, 20	4, 17, 13, 5, 11, 6, 40
V	[2, 6, 5]4, 48, 53, 20	3, 51, 24, 26, 40
VI	[2, 9, 40]44, 26, 40	3, 25, 35, 48, 8, 53, 2[0]
VII	[2, 12, 2]6, 40	2, 59, 47, 9, 37, 46[, 40]
VIII	[2, 15, 12, 3]5, 33, 20	2, 45, 2, 13,[20]
IX	[2, 16, 11,]6, 40	2, 40
X	[2, 13, 25, 11, 6],⌞40⌟	2, 40

Rev.

	Φ_1	G_1
S.E. 35, XI	[2, 10, 59, 4, 26, 40]	[3] 13, 2[4, 41, 28, 53, 20]
XII	[2, 13, 45]	[////]4, 24 [////////]
36, I	[2, 16, 30, 55, 33, 20]	2[//]23[////////////]
II	[2, 14, 52, 46, 40]	2, 40 blank
III	[2, 12, 6, 51]6, 40	2, 40, .9 [////////////]
IV	[2, 9, 20, 55, 3]3, 20	3, 5, 3[8, 1, 28, 53, 20]
V	[2, 6, 35] blank	3, 31, 26, 4[0]
VI	[2, 3, 49, 4]26, 40	3, 57, 15, 1[8, 31, 6, 40]
VII	[2, 1] 3, 8[53, 20]	[4, 23, 3, 57, 2, 13, 20]
VIII	[1, 58, 17] 13, 20	[//////////////////]
IX	[2, 0, 4]37, 46[40]	[//////////////////]
X	[2, 2]50[33, 20]	[4, 29, 24, 11, 51, 6, 40]
XI	[2, 5]⌜36, 28, 53, 20⌝	[4, 3, 35, 33, 20]
XII	[2, 8]22, 24,⌞26⌟[40]	[3, 37, 46, 54, 48, 53, 20]
37, I	[2, 11]⌞8⌟[20]	[3, 11, 58, 16, 17, 46, 40]

. = 𒑱

Text B: BM 36824.

Though M.B. 37222 joins B.M. 36824, its reverse is destroyed. No edges are preserved. At the end of obverse, line 14, a trace of a vertical ruling is preserved; after it the beginning of a sign is just visible. The sign is unidentifiable, but clearly not a number; thus the G-column was not followed by another Φ – G pair.

The reverse is in a wretched state, yet I feel reasonably confident of my restoration[14] since it was done without a view to dates, which turned out to match those of the obverse well. However, all readings that cannot be controlled by computations are very tentative.

The entries for which G differs from Ĝ, and most of which are at odds with the standard conversion rules, are collected with similar values from Texts **C** and **D** in Table 5.

Critical Apparatus.

Rev. 2. Reading very uncertain; there is even a possibility that the number may agree with

$$\hat{G} = 2,47,36,2,57,46,40$$

and so this entry is omitted from Table 5.

Rev. 5. As noted in the transcription, the G-value is written 2,40,.9, . . ., where "." represents the separation sign consisting of two diagonal wedges, to distinguish it from 2,49, . . .

Text C: B.M. 37203 (= 80-6-17, 956).
Contents: G_2 for S.E. 39, X to S.E. 40, IV.
Transcription: Table 3.
Photograph: Plate I.

TABLE 3.

		(Φ_2)	G_2
S.E. 39, X	1.	2, 7,22,35,33,20	3,4[7, 5,11, 6,40]
XI		2,10, 8,31, 6,40	3,21,[16,32,35,33,20]
XII		2,12,54,26,40	2,55,27,54, 4,⌜26⌝[40]
XII₂		2,15,40,22,13,20	2,41,12,11,15[//////]
40, I	5.	2,15,43,20	[2],40
II		2,12,57,24,26,40	[2],40,25,39, 7,13,20
III		2,10,11,28,53,20	[2,///]19,45,11, 6,40
IV		2, 7,25,33,20	[3,23,34]48,53,20

Text C: BM 37203.

[14] My restoration of these four texts was greatly facilitated by a table giving an entire number period of the functions Φ_1, Φ_2, $\hat{G}_1$, and $\hat{G}_2$. This table was executed by the Yale Computer according to the program of Miss VIVIAN REICH.

This text is a small flake. The photograph suggests that the left edge is preserved, but an unbiased inspection showed that the tablet rather was broken along a vertical ruling. I have supplied the corresponding values of Φ_2 in the transcription.

Text D: B.M. 37600 (= 80-6-17, 1357).
Contents: Φ_2, G_2 for S.E. 42, XII to S.E. 43, V.
Transcription: Table 4.
Photograph: Plate I.

The text is a small fragment with only one side, and no edges, preserved.

These four texts are unique in several respects. First it is clear that they are not fragments of standard System A ephemerides, even though the functions Φ and G progress in monthly steps. In the ephemerides, Φ and G are separated by several columns, while they are juxtaposed in Text **B** and **D**. Text **A** has no space for an initial column of dates, and Text **C** shows so many similarities to Text **D**, in its style of writing, in the dates, and in its deviations from the standard $\Phi - G$ scheme, that I should not be surprised if these two fragments came from the same tablet.

Second, the dates of these texts are very early; as said, my dating of them rests on the assumption that their Φ-values can be continued to the ACT material. The range of dates is from S.E. 6 to S.E. 43 (i.e., from 306 to 269 B.C.). The earliest lunar ephemeris in ACT (No. 1) is from S.E. 124, but an auxiliary table (No. 70), gives latitudes of full moons for at least S.E. 49 to 60.[14a]

Third, the values of G near its extrema differ from those derived from the standard ACT scheme. I have collected the aberrant pairs of values in Table 5

TABLE 4.

		Φ_2	G_2
S.E. 42, XII	1.	[1,59,32,35,33,20]	[//////]40 [////]
XII₂		[2, 2,18,31, 6,]40	⌜4⌝,34,23,12,35,[3,20]
43, I		[2, 5, 4,26]40	4, 8,34,34, 4,[6,40]
II		[2, 7,50,22]13,20	3,42,45,55,[3,20]
III	5.	[2,10,36,]7,46,40	3,16,57,17, 2,1[3,20]
IV		[2,13,22,]3,20	2,⌞53⌟,⌞7⌟,23,3[0]
V		[2,16, 8, 8,]3,20	[//////////////]

Text D: BM 37600.

[14a] See now the note added in proof at the end of this paper.

TABLE 5.

		Φ	G_{text}	G_{ACT}	$\hat{G}$
Text B, Obv.	14.	1,58,37, 2,13,20↑	4,56	4,56	5, 1, 3,42,13,20
	20.	2,15,12,35,33,20↑	2,45, 2,13,[20]	2,40,53,20	2,33,58,31, 6,40
	21.	2,16,11, 6,40 ↓	2,40	2,40	2, 8, 9,52,35,33,20
	22.	2,13,25,11, 6,40↓	2,40	2,40	2,27,38,16,17,46,40
Rev.	3.	2,16,30,55,33,20↑	2[//]23[//////]	2,40	2,21,47,24,26,40
	4.	2,14,52,46,40 ↓	2,40	2,40	2,14, 0,44,26,40
	5.	2,12, 6,51, 6,40↓	2,40,.9[//////]	2,43, 7,57,46,40	2,39,49,22,57,46,40
Text C	4.	2,15,40,22,13,20↑	2,41,12,11,15[33,20]	2,40, 7,46,40	2,29,39,15,33,20
	5.	2,15,43,20 ↓	2,40	2,40	2, 6, 8,53,20
	6.	2,12,57,24,26,40↓	[2]40,25,39, 7,13,20	2,40,27,24,26,40	2,31,57,31,51, 6,40
	7.	2,10,11,28,53,20↓	[2,///]19,45,11, 6,40	$G = \hat{G}$	2,57,46,10,22,13,20
Text D	1.	1,59,32,35,33,20↑	[//////]40 [////]	4,55,51, 6,40	5, 0,11,51, 6,40
	6.	2,13,22,13,20 ↓	2,⌞53⌟,⌞7⌟,23,3[0]	2,51,13,20	2,51, 8,38,31, 6,40

together with the G-values expected from ACT and the values of Ĝ. Most of the evidence concerns the situation near the minimum of G, and here the variants appear generally larger than the expected values. It is out of the question that the deviations are caused by faulty interpolation in the standard conversion table, for in one case (Text **B**, Obv. 20) the Φ-value is precisely in the ACT scheme, and in another (Text **C**, line 7) the Φ-value should imply that $G = \hat{G}$, which it is not.

The evidence is so scant and fragmentary that I could not comfortably reconstruct an underlying scheme, even assuming that the texts are consistent.

It is ironic, but perhaps to be expected, that this new evidence for variant $\Phi - G$ relations appears precisely at the moment when we have learned to control the standard scheme. Yet it confirms my feeling that Babylonian astronomical activities were more varied and diffuse than the ACT material would lead us to believe.

Text E

Text E: B.M. 36311 (= 80-6-17, 37) + B.M. 36593 (= 80-6-17, 321).
Contents: Φ, Λ, and X.
Transcription: Tables 6 and 7.
Photograph: Plates II and III.
Colophon: invocation in lower left corner of upper edge:

[*ina a-mat* ᵈe]n *u* ᵈgašan-*i̯a liš-lim*

= at the command of the deities Bēl and Bēlti, may it go well.

Description of Text.

This tablet is large; when unbroken it was about $6^1/_2$ inches wide and 7 inches high. The top edge and the right edge are almost complete; a small piece of the left edge remains, while the bottom edge is destroyed, though the restoration shows that only a few lines are missing. The four columns continue from the obverse over the bottom edge to the reverse. Horizontal alignment is carefully observed, and horizontal rulings are preserved. There were 62 lines to each side, the reverse spilling over onto the top edge.

The text is a copy, for the scribe wrote *ḫi-bi* (i.e., broken) thrice in the upper right corner. He could easily have restored the missing signs, had he so chosen, for the fourth column, as the third, is symmetrical about the bottom edge, so its beginning on the obverse is identical, in opposite order, with its end on the reverse, which is preserved.

At the end of Obverse II,1 after a number which must have been 2,13,20 one may read *šá* [*si-*]*man* (concerning the time). NEUGEBAUER has already drawn attention to a parallelism between the usage of "2,13,20" and of *si-man*.[15]

In the last column the scribe left an unusually large space between the initial 10 (or 11) and the remaining one or two digits. One might well wonder if the initial digits should be read with the others at all, were it not for the fact that the initial 10 changes to 11 precisely when required by the difference of 54 in the third digit (Obverse IV, 43–44).

Critical Apparatus.

Obv. II, 9. 2,10,57,*46*,40: *46* looks like 45.
Obv. II,16. [2],8,*53*,20: *53* looks like 52.
Obv. III,10. 3,44,5[3,20]: erasure between 44 and 53.
Obv. III,13. 3,*36*: *36* looks like 56.
Obv. III,42. [1,6,36],*17*,46,40: *17* damaged, might be read 45.
Obv. IV. *ḫi-bi* is written after the initial 10 in lines 3,4, and 6.
Rev. II,21. [. . .]6,40 should be 2,7,44,48,53,20; the scribe may have copied the corresponding number in III,21 in error.
Rev. IV, 2. 11,11,3[*9*]: a faint trace remains of what may have been a vertical wedge rather than the diagonal wedges expected for 9.

The separation sign consisting of two diagonal wedges, transcribed as ".", is used three times (Obv. IV,44, Rev. III,28, and Rev. III,35) to denote

[15] ACT I, p. 212; see also his Saros paper.

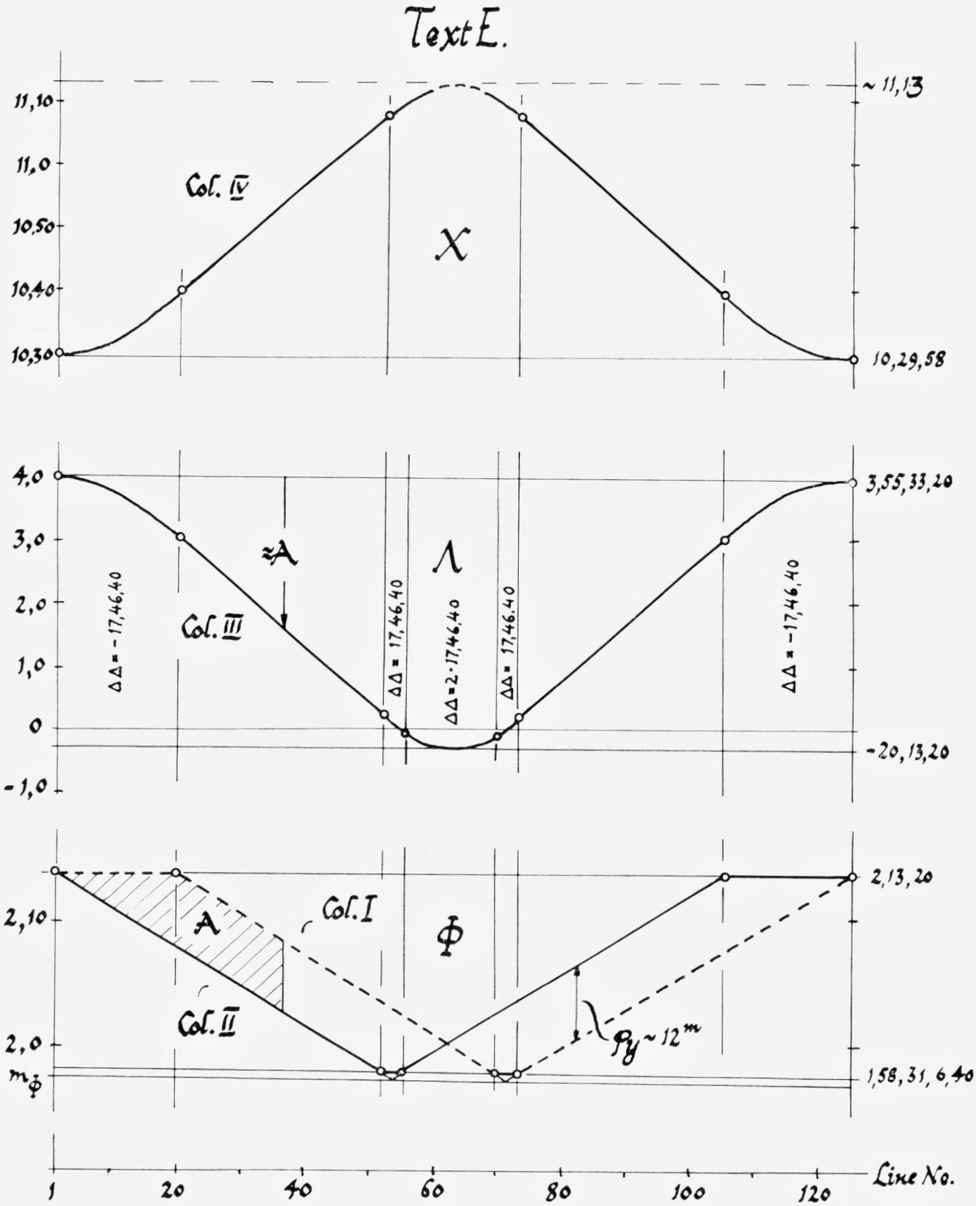
Text E.
~ 11,13
11,10
11,0
10,50
10,40
10,30
Col. IV
X
10,29,58
4,0
3,0
2,0
1,0
0
-1,0
zA
Λ
Col. III
ΔΔ = -17,46,40
ΔΔ = 17,46,40
ΔΔ = 2·17,46,40
ΔΔ = 17,46,40
ΔΔ = -17,46,40
3,55,33,20
-20,13,20
2,10
2,0
m_Φ
A
Col. I
Φ
Col. II
ρ_y ~ 12^m
2,13,20
1,58,31,6,40
1
20
40
60
80
100
120
Line No.

Fig. 4.

0, i.e., an empty sexagesimal place. In Rev.IV,56 and 57, the entries are written 10,30,.7 and 10,30,.[1] to distinguish them from 10,37 and 10,31, respectively.

Astronomical Commentary.

The first two columns of Text **E** concern Φ, the third Λ, which is known from ACT Nos. 207d and 207e, and the fourth X, as I call it, which is found here for the first time. Two parameters for Φ play a fundamental rôle in the relations between Φ and Λ. One is the increment of Φ from ascending branch to ascending branch corresponding to 12 months (one "year"):

$$\varphi_y = 12 \cdot d_\Phi - 2\,\Delta_\Phi = -0;5,22,35,33,20^{\mathrm{H}},$$

and the other is Φ's increment, again from ascending branch to ascending branch, corresponding to 223 months (one Saros):

$$\varphi_s = 223 \cdot d_\Phi - 16 \cdot 2\Delta_\Phi = -0;0,17,46,40,0^{\mathrm{H}}.$$

φ_s is a well-known parameter, and it plays a prominent rôle in the Saros text. φ_y was identified by NEUGEBAUER in ACT No. 207d, but its significance for Λ was not yet understood.

The uncovering of the table's structure proceeds from a crucial relation between the entries in the first three columns. The relation is, that the difference between the second and the first column is the line by line difference of the third, or more precisely (using $\mathrm{I}(n)$ to mean the entry in Column I, line n, and similarly for the other columns)

$$\mathrm{II}(n) - \mathrm{I}(n) = \mathrm{III}(n+1) - \mathrm{III}(n). \tag{3.1}$$

Relation (3.1) holds exactly throughout the text. (In modern language, Column III measures very nearly the area A between the graphs of Columns I and II – but for a constant – as suggested in Figure 4).

Now, on most of the obverse we find that the first two columns, both of which present Φ values, obey the relation

$$\mathrm{I}(n+1) - \mathrm{I}(n) = \mathrm{II}(n+1) - \mathrm{II}(n) = \varphi_s, \tag{3.2}$$

and that further

$$\mathrm{II}(n) - \mathrm{I}(n) = \varphi_y. \tag{3.3}$$

Thus, where (3.2) and (3.3) are valid, they imply

TABLE 6.

I	II	III	IV
[2,13,2]0	[2,13,20]	3,55,33,20	
[2,13,2]0	[2,13,2],13,20	3,55,33,20	
[2,13,2]0	2,[12,44],26,40	3,55,15,33,20	10,<30,1>
[2,13,2]0	2,12,26,40	3,54,40	10,<30,7>
[2,13,2]0	2,12,8,53,20	3,53,46,40	10,<30,16>
[2,13]20	2,11,51,6,40	3,52,35,33,20	10,<30,28>
[2,13]20	2,11,33,20	3,51,6,40	10,<30,43>
[2,13,20]	2,11,15,33,[20]	[3],49,20	10,31,1
[2,13,20]	2,10,57,46,40	[3],47,15,33,2[0]	10,31,24
[2,13,20]	2,10,40	3,44,5[3,20]	10,31,48
[2,13,20]	2,10,22,13,20	[3],42,1[3,20]	10,32,15
[2,13,20]	2,10,4,26,40	3,39,[15,33,20]	10,32,45
[2,13,20]	2,9,46,40	3,36	10,33,[1]8
[2,13,20]	[2],9,28,53,20	[3,32,26,40]	10,33,[54]
[2,13,20]	[2],9,11,6,40	[3,28,35,33],20	10,34,33
[2,13,20]	[2],8,53,20	[3,24,26,40]	10,35,15
[2,13,20]	[2],8,35,33,20	[3,20]	10,36
[2,13,20]	[2],8,17,46,40	[3,15,15,33],20	10,36,48
[2,13,20]	[2],8	3,1[0,13,20]	10,37,39
[2,13,4,48,53,20]	[3],7,42,13,20	3,4,[53,20]	10,38,33
[2,12,47,2,13,20]	[2],7,24,2[6],40	2,59,[30,44],26,40	10,39,27
[2,12,29,15,33,20]	[2,7],6,[40]	2,54,8,[8,5]3,20	10,40,21
[2,12,11,28,53,20]	[2],6,48,53,20	2,48,[45],33,20	10,41,15
[2,11,53,42,13,20]	[2],6,31,6,40	2,43,[22],57,46,40	10,42,9
[2,11,35,55,33,20]	[2],6,13,20	2,38,[0],22,13,20	10,43,3
[2,11,18,8,53,20]	[2,5],55,33,20	2,32,[3]7,46,40	10,43,57
[2,11,0,22,13,20]	[2,5,3]7,46,40	2,27,[15],11,6,40	10,44,51
[2,10,42,35,33,20]	[2,5,20]	2,21,5[2],35,33,20	10,45,45
[2,10,24,48,53,20]	[2,5,2,1]3,20	2,16,[30]	10,46,39
[2,10,7,2,13,20]	[2,4,44,2]6,40	2,11,[7,2]4,26,40	10,47,33
[2,9,49,15,33,20]	[2,4,26,40]	2,5,[44],48,53,20	10,48,27
[2,9,31,28,53,20]	[2,4,8,53],20	[2,0,22,1]3,20	10,49,21
[2,9,13,42,13,20]	[2,3,51,6,40]	[1,5]4,59,37,46,40	10,50,15
[2,8,55,55,33,20]	[2,3,33,20]	1,49,37,2,13,20	10,51,9
[2,8,38,8,53,20]	[2,3,15,33,20]	1,44,14,2[6],40	10,52,3
[2,8,20,22,13,20]	[2,2,57,46,40]	1,38,51,51,[6],40	10,52,57
[2,8,2,35,33,20]	[2,2,40]	1,33,29,15,33,20	10,53,51
[2,7,44,48,53,20]	[2,2,22,13,20]	1,28,6,40	10,54,45
[2,7,27,2,13,20]	[2,2,4,26,40]	[1,2]2,44,4,26,40	10,55,39
[2,7,9,15,33,20]	[2,1,46,40]	[1,17],21,28,53,20	[10],56,33
[2,6,51,28,53,20]	[2,1,28,53,20]	[1,11,5]8,53,2[0]	[10,5]7,27
[2,6,33,42,13,20]	[2,1,11,6,40]	[1,6,36],17,46,40	10,[5]8,21
[2,6,15,55,33,20]	[2,0,53,20]	[1,1,13,42,13],20	10,59,9
[2,5,58,8,53,20]	[2,0,35,33,20]	[5]5,5[1,6],40	11,•,9
[2,5,40,22,13,20]	[2,0,17,46,40]	[5]0,28,[31,6],40	11,1,3
[2,5,22,35,33,20]	[2]	[4]5,5,5[5,33],20	11,1,57
[2,5,4,48,53,20]	[1,59,42,13,20]	[3]9,43,20	11,2,51
[2,4,47,2,13,20]	[1,59,24,26,40]	[3]4,20,44,[26,40]	11,3,45
[2,4,29,15,33,20]	[1,59,6,40]	[2]8,58,8,[53,20]	11,4,39
[2,4,11,28,53,20]	[1,58,48,53,20]	23,35,[53,20]	11,5,33
[2,3,53,42,13,20]	[1,58,31,6,40]	18,12,5[7,46,40]	11,6,27
[2,3,35,55,33,20]	[1,58,31,6,40]	[12,50,22,13,20]	11,7,21
[2,3,18,8,53,20]	[1,58,31,6,40]	[7,45,33,20]	[11],8,12,
[2,3,0,22,13,20]	[1,58,31,6,40]	[2,58,31,6,40 tab]	[11,9]
[2,2,42,35,33,20]	[1,58,31,6,40]	[1,30,44,26,40 lal]	[11,9,45]
[2,2,24,48,53,20]	[1,58,33,42,13,20]	[5,42,13,20]	[11,10,27]
[2,2,7,2,13,20]	[1,58,51,28,53,20]	[9,33,20]	[11,11,6]
[2,1,49,15,33,20]	[1,59,9,15,33,20]	[12,48,53,20]	[11,11,32]
[2,1,31,28,53,20]	[1,59,27,2,13,20]	[15,28,53,20]	destroyed
[2,1,13,42,13,20]	[1,59,44,48,53,20]	[17,33,20]	
[2,0,55,55,33,20]	[2,0,2,35,33,20]	[19,2,13,20]	
[2,0,38,8,53,20]	[2,0,20,22,13,20]	[19,55,33,20]	

Text E, Obv.

TABLE 7.

I	II	III	IV
[2, 0, 20, 22, 13, 20]	[2, 0, 38, 8, 53, 20]	[20, 13, 20]	destroyed
[2, 0, 2, 35, 33, 20]	[2, 0, 55, 55, 33, 20]	[19, 55, 33, 20]	
[1, 59, 44, 48, 53, 20]	[2, 1, 13, 42, 13, 20]	[19, 2, 13, 20]	
[1, 59, 27, 2, 13, 20]	[2, 1, 31, 28, 53, 20]	[17, 33, 20]	
[1, 59, 9, 15, 33, 20]	[2, 1, 49, 15, 33, 20]	15, 28, 5[3, 20]	[///////]
[1, 58, 51, 28, 53, 20]	[2, 2, 7, 2, 13, 20]	12, 48, 53 [20]	11, 11, ⌈3⌉[9]
[1, 58, 33, 42, 13, 20]	[2, 2, 24, 48, 53, 20]	9, 33, 20	[11], 11, 6
[1, 58, 31, 6, 40]	[2, 2, 42, 35, 33, 20]	5, 42, 1[3, 20]	11, 10, 27
[1, 58, 31, 6, 40]	[2, 3, 0, 22, 13, 20]	1, 30 [44, 26, 40] ⌈lal⌉	11, 9, 45
[1, 58, 31, 6, 40]	[2, 3, 18, 8, 53, 20]	2, 58 [31, 6], 40 tab	11, 9
[1, 58, 31, 6, 40]	[2, 3, 35], 55 [3]3, 20	7, 4[5, 33], 20	11, 8, 12
[1, 58, 31, 6, 40]	[2, 3, 53] ⌈42⌉, 13, 20	12 [50, 22, 1]3, 20	11, 7, 21
[1, 58, 48, 53, 20]	[2, 4, 11], 28, 53, 20	1[8, 12, 5]7, 46, 40	11, 6, 27
[1, 59, 6, 40]	[2, 4, 2]9, 15, 33, 20	2[3, 3]5, 33, 20	11, 5, 33
[1, 59, 24, 26, 40]	[2, 4, 4]7, 2, 13, 20	[28], 58, 8, 53, 20	11, 4, 39
[1, 59, 42, 13, 20]	[2, 5, 4, 48, 53, 20	34, 20, 44, 26, 40	11, 3, 45
[2]	[2, 5, 22], 35, 3[3, 20]	39, 43, 20	11, 2, 51
[2, 0, 17, 46, 40]	[2, 5, 40, 2]2, 13, 20	45, 5, 55, 33, 20	11, 1, 57
[2, 0, 35, 33, 20]	[2, 5, 58, 8, 5]3, 20	50, 28, 31, 6, 40	1[1, 1], 3
[2, 0, 53, 20]	[2, 6, 15, 55, 3]3, 20	55, 51, 6, 40	[11, 0], 9
[2, 1, 11, 6, 40]	[2, 6, 33, 42], 13, 20	1, 1, 13, 42, 13, 20	10, 59, 15
[2, 1, 28, 53, 20]	[2, 6, 51, 28], 53, 20	1, 6, 36, 17, 46, 40	10, 58, 21
[2, 1, 46, 40]	[2, 7, 19, 15], 33, 20	1, 11, 58, 53, 20	10, 57, 27
[2, 2, 4, 26, 40]	[2, 7, 27, 2], 13, 20	1, 17, 21, 28, 53, 20	10, 56, 33
[2, 2, 22, 13, 20]	[///////]6, 40	1, 22, 44, 4, 26, 40	10, 55, 39
[2, 2, 40]	[2, 8, 2] 35, 33, 20	1, 28, 6, 40	10, 54, 45
[2, 2, 57, 46, 40]	[2, 8, 20] 22, 13, 20	1, 33, 29, 15, 33, 20	10, 53, 51
[2, 3, 15, 33, 20]	[2, 8, 3]8, 8, 53, 20	1, 38, 51, 51, 6, 40	10, 52, 57
[2, 3, 33, 20]	[2, 8], 55, 55, 33, 20	1, 44, 14, 26, 40	10, 52, 3
[2, 3, 51, 6, 40]	[2, 9, 1]3, 42, 13, 20	1, 49, 37, 2, 13, 20	10, 51, 9
[2, 4, 8, 53, 20]	[2, 9, 3]1, 28, 53, 20	1, 54, 59, 37, 46, 40	10, 50, 15
[2, 4, 26, 40]	[2, 9, 49], 15, 33, 20	2, ., 22, 13, 20	10, 49, 21
[2, 4, 44, 26, 40]	[2, 10], 7, 2, 13, 20	2, 5, 44, 48, 53, 20	10, 48, 27
[2, 5, 2, 13, 20]	[2, 10, 24], 48, 53, 20	2, 11, 7, 24, 26, 40	10, 47, 33
[2, 5, 20]	[2, 10, 42, 3]5, 33, 20	2, 16, 30	10, 46, 39
[2, 5, 37] ⌈46⌉ [40]	[2, 11, 0, 22, 1]3, 20	2, 21, 52, 35, 33, 20	10, 45, 45
[2, 5, 5]5, 33 [20]	2, 11, 18, 8, 53, 20	2, 27, 15, 11, 6, 40	10, 44, 51
[2, 6], 13, 20	2, 11, 35, 55, 33, 20	2, 32, 37, 26, 40	10, 43, 57
[2] 6, 31, 6 [40]	2, 11, 53, 42, 13, 20	2, 38, ., 22, 13, 20	10, 43, 3
[2, 6] 48, 53, [20]	2, 12, 11, 28, 53 [20]	2, 43, 22, 57, 46, 40	10, 42, 9
[2], 7, 6, 40	2, 12, 29, 15, 33, 20	2, 48, 45, 33, 20	10, 41, 15
2, 7, 24, 26 [40]	[2], 12, 47, 2, 13, 20	2, 54, 8, 8, 53, 20	10, 40, 21
2, 7, 42, 13, [20]	[2], 13, 4, 48, 53, 20	2, 59, 3[0], 44, 26, 40	10, 39, 27
2, 8	[2], 13, 20	[3], 4, 53, 20	10, 38, 33
2, 8, 17 [46, 40]	2, 13, 20	⌈3⌉, 10, 13, 20	10, 37, 39
2, 8, 35, [33], 20	2, 13, 20	3, 15, 15, 33, 20	10, 36, 48
2, 8, 53, [20]	2, 13, 20	3, 20	10, 36
2, 9, 11, [6, 40]	2, 13, 20	3, 24, 26, 40	10, 35, 15
2, 9, 28, 53, 20	2, 13, 20	3, 28, 35, 33, 20	10, 34, 33
2, 9, 46, 40	2, 13, 20	3, 32, 26, 40	10, 33, 54
2, 10, .4, 26, 40	2, 13, 20	3, 36	10, 33, 18
2, 10, 22, 13, 20	2, 13, 20	3, 39, 15, 33, 20	10, 32, 45
2, 10, 40	2 [13] 20	3, 42, 13, 20	10, 32, 15
2, 10, 57, 46, 40	2 [13] 20	3, 4[4], 53, 20	10, 31, 48
[2], 11, 15, 33, 20	2 [13, 20]	3, 47, 15, 33, 20	10, 31, 24
[2, 11] 33, 20	[2, 13, 20]	3, 49, 20	10, 31, 1
[2, 11, 51], 6, 40	[2, 13, 2]0	3, 51, 6, 40	10, 30, 43
[2, 12] 8, 53, 20	[2, 13]20	3, 52, 35, 33, 20	10, 30, 28
[2, 12, 2]6, 40	[2]13, 20	3, 53, 46, 40	10, 30, 16
[2, 12, 44, 2]6, 40	[2]13, 20	3, 54, 40	10, 30, .7
[2, 13, 2], 13, 20	[2]13, 20	3, 55, 15, 33, 20	10, 30, .[1]
[2, 13, 2]0	[2]13, 20	3, 55, 33, 20	10 [2]9, 5[8]

Text E, Rev.

(i) that to move one line down means to advance 223 months, and
(ii) that to go horizontally from Column I to Column II means to advance 12 months.

Further, if we allow for truncation of Φ at 2,13,20 and 1,58,31,6,40 and for reflexion in M_Φ and m_Φ, it is readily shown that (i) and (ii) hold throughout the text.

The relation (3.1) therefore states that the difference in Λ for 223 months is the same as the difference in Φ for 12 months. It is therefore possible to

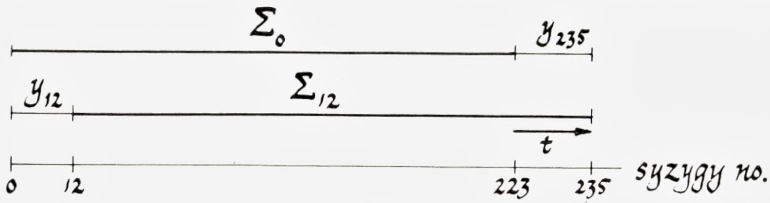

Fig. 5.

give an argument completely similar to the one above (p. 5) for Φ and G showing first, that Λ is measured in large hours and second, that the difference between the length of 12 months and Λ is constant.

First, we learn from ACT Nos. 207d and 207e that the Λ-value given in III(n) is associated with the Φ-value in II(n) and not with that in I(n). Now we know that the value of G associated with a given syzygy refers to the preceding month, so the derivation of the $\Phi-$G scheme to be given later shows that the value of Φ associated with a syzygy measures the following Saros (as we have already used). We shall then show that the difference between the value of Λ associated with a given syzygy (via a Φ-value) differs by a constant from the length of the preceding 12 months.

Let Σ_{12} be a Saros beginning 12 months after the Saros Σ_0; let further y_{235} be the length of the 12 months following Σ_0 and y_{12} be the length of the 12 months preceding Σ_{12}. We then have (see Figure 5)

$$\Sigma_{12}-\Sigma_0 = y_{235}-y_0 .$$

But we have, that

$$\Sigma_{12}-\Sigma_0 = \Phi_{12}-\Phi_0 ,$$

so since, (as shown by the structure of our tablet and the association of Λ with syzygies as in ACT)

$$\Phi_{12}-\Phi_0 = \Lambda_{235}-\Lambda_{12}$$

we have

$$y_{235} - y_{12} = \Lambda_{235} - \Lambda_{12}$$

or

$$y - \Lambda = \text{constant.}$$

The values of Λ are such that it appears reasonable that Λ is the excess in large hours of 12 months over a whole number of days; I shall return to this point below after discussing the Φ – G relations further.

Λ may be negative. In the text, only the situation where Λ goes through zero increasingly is preserved (Rev. III, 5 and 6); the last negative value is followed by "lal", and the first positive by "tab".

In Table 8, I have displayed the structure of the first three columns of Text **E**. A few remarks suffice to show how it (or rather the table in ACT consisting in essence of Columns II and Λ) was constructed. First, both φ_y and φ_s are negative when Φ goes from one *increasing* branch to another; it is therefore clear, that the Φ-values in Column II must belong to increasing branches until line 54; thus I have designated them with arrows pointing upwards.

Second, we recall that the values of Λ are associated with the values of Φ given in Column II of this text.

The table is readily constructed from (i) and (ii) above. We begin in Column II with the value 2,13,20↑. The corresponding value in Column I is found by subtracting φ_y from 2,13,20 and reflecting in M_Φ (φ_s is negative). We obtain the value given in the parenthesis of Column I, line 1, of Table 8; since this is larger than 2,13,20 the effective value of I(1) is 2,13,20. Columns I and II now proceed with a difference per line of φ_s, supplied with the appropriate sign; where values exceed 2,13,20, we write 2,13,20. As long as Column I remains constant at 2,13,20, while Column II increases by φ_s per line, the quantity

$$\mathrm{II}(n) - \mathrm{I}(n) = \Delta\Lambda$$

will decrease by 0;0,17,46,40 per line, i.e., the second difference of Λ is constant and equal to φ_s.

From line 20 to line 50, Columns I and II run parallel, so $\Delta\Lambda$ is constant and equal to φ_y.

In line 51, Column II reaches the value 1,58,31,6,40. If we proceed with the difference of φ_s, and reflect in m_Φ, we get the values given in the parentheses. As long as these are below 1,58,31,6,40, Column II remains constant at this value. This means, that $\Delta\Lambda$ increases by 0;0,17,46,40 per line, or that $\Delta\Delta\Lambda$ is constant and equal to $-\varphi_s$. From line 56, Column I decreases by

TABLE 8.

	I	II	II − I = ΔΛ	Λ	
1.	2,13,20 (2,15,27,2,13,20)	2,13,20 ↑	0	3,55,33,20	ΔΔΛ = −17,46,40
	2,13,20 (2,15,44,48,53,20)	2,13,2,13,20 ↑	− 17,46,40	3,55,33,20	
	2,13,20 (2,16,2,35,33,20)	2,12,44,26,40 ↑	− 35,33,20	3,55,15,33,20	
	2,13,20 (2,16,20,22,13,20)	2,12,26,40 ↑	− 53,20	3,54,40	
5.	2,13,20 (2,16,38,8,53,20)	2,12,8,53,20 ↑	− 1,11,6,40	3,53,46,40	
	2,13,20 (2,16,55,55,33,20)	2,11,51,6,40 ↑	− 1,28,53,20	3,52,35,33,20	
	2,13,20 (2,16,55,55,33,20)	2,11,33,20 ↑	− 1,46,40	3,51,6,40	
	2,13,20 (2,16,38,8,53,20)	2,11,15,33,20 ↑	− 2,4,26,40	3,49,20	
	2,13,20 (2,16,20,22,13,20)	2,10,57,46,40 ↑	− 2,22,13,20	3,47,15,33,20	
10.	2,13,20 (2,16,2,35,33,20)	2,10,40 ↑	− 2,40	3,44,53,20	
	2,13,20 (2,15,44,48,53,20)	2,10,22,13,20 ↑	− 2,57,46,40	3,42,13,20	
	2,13,20 (2,15,27,2,13,20)	2,10,4,26,40 ↑	− 3,15,33,20	3,39,15,33,20	
	2,13,20 (2,15,9,15,33,20)	2,9,46,40 ↑	− 3,33,20	3,36	
	2,13,20 (2,14,51,28,53,20)	2,9,28,53,20 ↑	− 3,51,6,40	3,32,26,40	
15.	2,13,20 (2,14,33,42,13,20)	2,9,11,6,40 ↑	− 4,8,53,20	3,28,35,33,20	
	2,13,20 (2,14,15,55,33,20)	2,8,53,20 ↑	− 4,26,40	3,24,26,40	
	2,13,20 (2,13,58,8,53,20)	2,8,35,33,20 ↑	− 4,44,26,40	3,20	
	2,13,20 (2,13,40,22,13,20)	2,8,17,46,40 ↑	− 5,2,13,20	3,15,15,33,20	
	2,13,20 (2,13,22,35,33,20)	2,8 ↑	− 5,20	3,10,13,20	
20.	2,13,4,48,53,20	2,7,42,13,20 ↑	− 5,22,35,33,20	3,4,53,20	
	2,12,47,2,13,20	2,7,24,26,40 ↑	− 5,22,35,33,20	2,59,30,44,26,40	
	⋮	⋮	⋮	⋮	
50.	2,4,11,28,53,20	1,58,48,53,20 ↑	− 5,22,35,33,20	+ 23,35,33,20	ΔΔΛ = 17,46,40
	2,3,53,42,13,20	1,58,31,6,40 ↑	− 5,22,35,33,20	+ 18,12,57,46,40	
	2,3,35,55,33,20	1,58,31,6,40 (1,58,13,20) ↑	− 5,4,48,53,20	+ 12,50,22,13,20	
	2,3,18,8,53,20	1,58,31,6,40 (1,57,55,33,20) ↑	− 4,47,2,13,20	+ 7,45,33,20	
	2,3,0,22,13,20	1,58,31,6,40 (1,57,58,8,53,20) ↓	− 4,29,15,33,20	+ 2,58,31,6,40	
55.	2,2,42,35,33,20	1,58,31,6,40 (1,58,15,55,33,20) ↓	− 4,11,28,53,20	− 1,30,44,26,40	ΔΔΛ = 2·17,46,40
	2,2,24,48,53,20	1,58,33,42,13,20 ↓	− 3,51,6,40	− 5,42,13,20	
	2,2,7,2,13,20	1,58,51,28,53,20 ↓	− 3,15,33,20	− 9,33,20	
	2,1,49,15,33,20	1,59,9,15,33,20 ↓	− 2,40	− 12,48,53,20	
	2,1,31,28,53,20	1,59,27,2,13,20 ↓	− 2,4,26,40	− 15,28,53,20	
60.	2,1,13,42,13,20	1,59,44,48,53,20 ↓	− 1,28,53,20	− 17,33,20	
	2,0,55,55,33,20	2,0,2,35,33,20 ↓	− 53,20	− 19,2,13,20	
	2,0,38,8,53,20	2,0,20,22,13,20 ↓	− 17,46,40	− 19,55,33,20	
	2,0,20,22,13,20	2,0,38,8,53,20 ↓	+ 17,46,40	− 20,13,20	

0;0,17,46,40, while Column II increases by the same amount per line. Thus $\Delta\Delta\Lambda$ is constant, and equal to $-2\varphi_s$.

Λ is therefore completely determined from Φ and its initial value 3;55,33,20.

φ_s and φ_y are so related, that Columns I and II are symmetrical in the sense that Column II as a whole, counting both obverse and reverse, is the same as Column I reversed. Thus Λ is symmetrical about the first line of the reverse. It may also be noted that in Column I of Table 8 the values in the parentheses are symmetrical about M_Φ.

Because of these symmetry properties, one side of the text suffices to display the entire scheme. Thus I have given only the obverse (plus one line) in Table 8; the presence of the first half of Column I compensates for the absence of the last half of Column II. From Column II, line 63, we go to Column I, line 61. Two changes should be observed, first, that the Φ-value now belongs to a descending branch, and second, that the Λ-value associated with a Φ-value in Column I is now found one line lower. These properties of the $\Phi - \Lambda$ scheme may well have some bearing on the structure of the actual Φ – G scheme, as we shall see below.

Before I finish the discussion of Λ, it is well to consider the Φ – G relations, where one may readily apply the technique displayed in Text **E** with but slight and obvious modifications. This I have done in Table 9.[15a] The fundamental relation is, that the monthly difference in Φ (the Saros) is the same as the "Sarosly" difference in G (the month). Columns A and B contain Φ-values; the effective values are truncated at 2,13,20 above and 1,58,31,6,40 below, while the untruncated values are given in parentheses. The parameter which replaces φ_y is

$$\varphi_m = d_\Phi = 0;2,45,55,33,20,$$

i.e., the monthly difference in Φ on an ascending branch. As before, the line by line difference for both Column A and Column B is φ_s (with appropriate sign) except where modified by necessary reflexions and truncations. Thus, to move horizontally from Column A to Column B is to advance one month in time, and to move one line down corresponds to 223 months. If n denotes the line number, the table is constructed so that

$$\mathrm{G}(n+1) - \mathrm{G}(n) = \mathrm{B}(n) - \mathrm{A}(n),$$

which simply expresses the fundamental relation.

Thus G is in principle completely determined from Φ and initial value.

There is excellent, though not complete, agreement between this theoretical scheme and the one extracted from procedure texts and ephemerides in ACT. More precisely, the constructed scheme deviates in two respects from that in ACT. The first anomaly is, that in four instances (lines 57–60) the generated

[15a] ACT No. 207 cd may be a small fragment of just such a table.

Table 9.

	A	B		B − A = ΔG	G	ACT sic.
1.	2,13,20	2,13,20	(2,16,5,55,33,20↑)	0	2,40	
	2,13,2,13,20	2,13,20	(2,15,48,8,53,20↑)	17,46,40	2,40	
	2,12,44,26,40	2,13,20	(2,15,30,22,13,20↑)	35,33,20	2,40,17,46,40	
	2,12,26,40	2,13,20	(2,15,12,35,33,20↑)	53,20	2,40,53,20	
5.	2,12,8,53,20	2,13,20	(2,14,54,48,53,20↑)	1,11,6,40	2,41,46,40	
	2,11,51,6,40	2,13,20	(2,14,37,2,13,20↑)	1,28,53,20	2,42,57,46,40	
	2,11,33,20	2,13,20	(2,14,19,15,33,20↑)	1,46,40	2,44,26,40	
	2,11,15,33,20	2,13,20	(2,14,1,28,53,20↑)	2,4,26,40	2,46,13,20	
	2,10,57,46,40	2,13,20	(2,13,43,42,13,20↑)	2,22,13,20	2,48,17,46,40	
10.	2,10,40	2,13,20	(2,13,25,55,33,20↑)	2,40	2,50,40	
	2,10,22,13,20	2,13,8,8,53,20↑		2,45,55,33,20	2,53,20	
	2,10,4,26,40	2,12,32,35,33,20↑		2,45,55,33,20	2,56,5,55,33,20	
	ΔA = −17,46,40	ΔB = −17,46,40				
50.	1,58,48,53,20	2,1,34,48,53,20↑		2,45,55,33,20	4,41,11,6,40	
	1,58,31,6,40	2,1,17,2,13,20↑		2,45,55,33,20	4,43,57,2,13,20	
	1,58,31,6,40 (1,58,13,20)	2,0,59,15,33,20↑		2,28,8,53,20	4,46,42,57,46,40	
	1,58,31,6,40 (1,57,55,33,20)	2,0,41,28,53,20↑		2,10,22,13,20	4,49,11,6,40	
	1,58,31,6,40 (1,57,58,8,53,20)	2,0,23,42,13,20↑		1,52,35,33,20	4,51,21,28,53,20	
55.	1,58,31,6,40 (1,58,15,55,33,20)	2,0,5,55,33,20↑		1,34,48,53,20	4,53,14,4,26,40	
	1,58,33,42,13,20	1,59,48,8,53,20↑		1,14,26,40	4,54,48,53,20	
	1,58,51,28,53,20	1,59,30,22,13,20↑		38,53,20	4,56,3,20	(4,56)
	1,59,9,15,33,20	1,59,12,35,33,20↑		3,20	4,56,42,13,20	(4,56,35,33,20)
	1,59,27,2,13,20	1,58,54,48,53,20↑		− 32,13,20	4,56,45,33,20	(4,56,35,33,20)
60.	1,59,44,48,53,20	1,58,37,2,13,20↑		− 1,7,46,40	4,56,13,20	(4,56)
	2,0,2,35,33,20	(1,58,29,15,33,20)				

G-values differ from the actual ones from ACT; I have appended the values from ACT in parentheses.

As we learn from the texts, a value of G is associated with the Φ-value in the same line in Column B, which reflects that while the value of G assigned to a given syzygy refers to the preceding month, the corresponding Φ-value measures the subsequent Saros. Thus, strictly speaking, Table 9 generates only half of the scheme in ACT; it assigns only G-values to the Φ-values in Column B, which all lie on an ascending branch. However, and this is the second anomaly, a comparison shows that this table nonetheless serves to represent the entire ACT scheme if one observes the following rule: to a Φ-value in line n of Column A, but belonging to the opposite branch (i.e.

descending, for the greater part), is assigned the G-value in line $(n+1)$. This is precisely what we should have expected if Table 9, were it continued, had turned out to be symmetrical in the same senses as the table for Λ, but that is not so. The reason is, of course, that φ_m and φ_s do not bear to each other as pleasant a relation as do φ_y and φ_s.

It is, therefore, very tempting to believe, that the Φ – G scheme, though for the greater part generated in a strict fashion, is modelled in its entirely after the $\Phi - \Lambda$ scheme. The first anomaly – the four aberrant values near the maximum – would then be the result of an attempt to symmetrise the G-values (it may be noted, that the actual differences in G are simple combinations of the expected ones). The second anomaly would then simply constitute the justification of the first.

It is natural to search for a direct connexion between Λ and G. The result is disappointing. First, if for a given value of Φ one compares the corresponding value of Λ with the sum of the appropriate 12 values of G, one does not get exact agreement modulo a whole number of days. Next, it is reasonable to seek a corollation between the initial values of Λ and G, and I believe they are extrema. I therefore took 12 consecutive monthly G-values, symmetrically disposed about G's maximum, and similarly 12 about its minimum, added them, and obtained

$$\sum_{i=1}^{12} (29^{d} + G_i) = 355^{d} + 3;32,21,43,42,13,20^{H} \qquad (i)$$

and

$$\sum_{i=1}^{12} (29^{d} + G_i) = 355^{d} - 0;43,20,59,15,33,20^{H}. \qquad (ii)$$

These two values must be the extrema of sums of the duration of 12 consecutive months, or very nearly so. The two values, $3;32,\ldots^{H}$ and $-0;43,\ldots^{H}$, fall short of the respective extrema of Λ by about $0;23^{H}$. Still, the difference between (i) and (ii) is very close indeed to the amplitude of Λ, but then that is only to be expected since both G and Λ are completely determined by Φ but for an additive constant, if we ignore the slight adjustments of G near its maximum.

I can therefore safely conclude that if I am right in identifying Λ with the excess of 12 months over a whole number of days, then the initial values of G and Λ are chosen independently of each other. Further, the purpose of Λ is clearly not to serve as a check for G.

For this reason, corroborated by the analogous situation for Φ and G, the high values of G are no obstacle to assuming that Λ is the excess of 12 months, not over 355^d, but rather over 354^d, i.e.,

$$12 \text{ months} = 354^d + \Lambda^H.$$

Indeed, 12 mean synodic months amount to about

$$354^d + 2;12, \ldots{}^H,$$

and 2;12, . . . is not far from a reasonable mean value of Λ. Further, the value 354 (but not 355) opens a possibility for interpreting Column X, as we shall see directly.

Column X, the fourth column of Text **E**, is in structure very like Column Λ (see Figure 4). The function is symmetrical in the same sense as Λ; near its extrema it has sections with constant second difference (of ± 3 and, if my reconstruction is right, in part of ± 6), and in between a linear stretch (with constant difference of ± 54). The non-linear parts of X near its extrema are shown separately in Tables 10 and 11.

Table 10 exhibits one irregularity of X near its minimum. The second difference, here usually 3, is once 1 and then 5 whereupon it becomes 3 again. In the appended columns I have displayed what X would be if the second difference remained constant at 3. The resulting minimum is more pleasant, viz., 10,30,0 versus 10,29,58.

Unfortunately, the maximum of X and its neighbouring values are destroyed. Thus the values in Table 11 are largely restored. My restoration rests on the assumption of a doubling (to 6) of the second difference in analogy with the case of Λ. This assumption is plausible, for it generates the correct number of lines and makes X symmetrical about its maximum.

X appears here for the first time. I believe that it can be identified as the epact measured in days corresponding to Λ, i.e., the variable difference between a constant year and the variable length of 12 months.

Indeed, Figure 4 suggests that X is a complement to Λ; more precisely, if Λ is converted into days and added to X interpreted as days (so that, e.g., its maximum is $11;12,54^d$) this sum, i.e.,

$$\tfrac{1}{6}\Lambda + X,$$

is very nearly constant throughout the text. Assuming my reconstructed extrema for X, these values all lie between $11;9,15,33,20^d$ and $11;9,31,46,40^d$. Interpreting Λ as above, we get:

TABLE 10.

	X	ΔX	$\Delta\Delta X$	X if constant $\Delta\Delta$	ΔX	$\Delta\Delta X$
Obv.21 = Rev.39	10,39,27					
	10,38,33	-54				
	10,37,39	-54	0			
	10,36,48	-51	3			
	10,36	-48	3			
	10,35,15	-45	3			
	10,34,33	-42	3			
	10,33,54	-39	3			
	10,33,18	-36	3			
	10,32,45	-33	3			
	10,32,15	-30	3			
	10,31,48	-27	3	10,31,48		
	10,31,24	-24	3	10,31,24	-24	
	10,31,1	-23	1	10,31,3	-21	3
	10,30,43	-18	5	10,30,45	-18	3
	10,30,28	-15	3	10,30,30	-15	3
	10,30,16	-12	3	10,30,18	-12	3
	10,30,7	-9	3	10,30,9	-9	3
	10,30,1	-6	3	10,30,3	-6	3
	10,29,58	-3	3	10,30	-3	3

TABLE 11.

	X	ΔX	$\Delta\Delta X$
Obv.50 = Rev.10	11,5,33		
	11,6,27	54	
	11,7,21	54	0
	11,8,12	51	-3
	11,9	48	-3
	11,9,45	45	-3
	11,10,27	42	-3
	11,11,6	39	-3
	11,11,39	33	-6
	11,12,6	27	-6
	11,12,27	21	-6
	11,12,42	15	-6
	11,12,51	9	-6
	11,12,54	3	-6
	11,12,51	-3	-6

$$12^{\mathrm{m}} + X = 354^{\mathrm{d}} + \tfrac{1}{6}\Lambda + X = 365;9,\ldots^{\mathrm{d}}$$

which is very near a value for the year, though too small. However, the text B.M. 36712[16] gives evidence for a year length of $6,5;10^{\mathrm{d}}$.

I believe that the slight variation of

$$\varepsilon = \tfrac{1}{6}\Lambda + X$$

is the result of rounding, not of X's values, but of its parameters. On the linear stretch, constancy of the quantity ε would require that ΔX, but for its sign, should be

$$\tfrac{1}{6}\Delta\Lambda = 0;0,53,45,55,33,20$$

while we find

$$\Delta X = 0;0,54,$$

and on the stretch of constant second difference, $\Delta\Delta X$ should be

$$\tfrac{1}{6}\Delta\Delta\Lambda = 0;0,2,57,46,40$$

while in fact

[16] A. SACHS and O. NEUGEBAUER, *A Procedure Text Concerning Solar and Lunar Motion: B. M. 36712*. Journal of Cuneiform Studies X, 1956, pp. 131–135.

$$\Delta\Delta X = 0;0,3,$$

and analogously where the second difference is doubled. This serves further to explain, that the transitions from constant second to constant first difference do not happen exactly in the same lines for Δ and for X. Once the decision to limit the number of sexagesimal places of X was made, the handsome arithmetical structure of X must have been deemed more important than the constancy of the generated year length.

Text F

Text F: B.M. 36775 (= 80-6-17, 512).
Contents: Procedure text for variants of Column F.
Photograph: Plate IV.

Description of Text.

This tablet is about 4 inches wide and $2^1/_2$ inches high. Not much clay is missing, but the surface is damaged, particularly that of the obverse. The writing is divided into sections by horizontal rulings. Traces of a few signs are left here and there in the destroyed section; thus the obverse clearly did not begin with line 1'.

Transcription.

Obverse

Section 1.

1'. [. . . .] 15 lal [. . . . .] 15 t[a]
2'. [. . . .] 2,45 zi-*m*[*a*. . . . .] t[a. . . . .] *ki-i al* 11,15
3'. [. . . .] tab-*ma* 2,11[. . . . .] 2,15 gar-*an*

Section 2.

4'. [. . . . . .] ⌊ta⌋ [. . . . .]46,52,30
5'. [. . . . . *gab-bi šá al-la*] 15 dir 15 e-*šú*
6'. [*mim-ma šá al* 11,15 lal-*ú* 11,15 e-*šú* 42 *šá*] itu 1,21,39,22,30
7'. [*šá* mu 2,20,37,30 *šá* 14 itu 23,15 *šá* 9 mu-m]eš 4,30 *šá* 18 mu-meš
8'. [. . . . .] zi blank gal (?)
9'. [. . . . .] blank

Section 3.

10'. [. . . . .] 11,15 zi
11'. [. . . . .] an ta 15 ta
12'. [. . . . . .] gar-*an ki-i al* 15 dir
13'. [. . . . . . .] gar-*an ki-i al* 15 lal 2,⌊14⌋ [. . . .]

Reverse

1. [. . . . . .] 13,5,9,22,30 [. . . .]
2. [. . . . . .] DU (?) 11,⌈15⌉ lal (?) 42,11,15 [. . . .]

Section 4.

3. [t]a 15,44,31,52,30 en 10,58,23,26,15 itu
4. [*ana* itu] 42,11,15 tab *u* lal *lìb-bu-ú šá* itu-meš *ta-ba-nu-ú*
5. [. .] *gab-bi šá al-la* 15 dir 15 e-*šú mim-ma šá al* 11,15
6. [lal-]*ú* 11,15 e-*šú* 42,11,15 *šá* itu 1,22,1,52,30 *šá* mu
7. [2,] 20,37,30 *šá* 14 itu 23,26(,15) *šá* 9 mu 4,41,15 *šá* 18
8. [mu] 4,11 *šal-meš* zi-*šú* kur-*ád* ta [1]5,4⌊4,31,52,30⌋
9. [en 10,]50,23,26,15 4,54,[8,26,15 . . . .] BAL (?) meš sar

Section 5.

10. [*ana tar-ṣa* 2,1]3,20 15 zi *ana tar-ṣa* [1,58,31,6,40 11,15 zi]
11. [ta 15,56],54,22,30 en [11,4,4,41,15 . . . . . .]

Rest destroyed.

Critical Apparatus.

Obv. 2'.]2,45 : 3,45 certainly possible

Rev. 3. 10,*58*,23,26,15 : error for 10,*50*,23,26,15. The correct value, but for the initial 10, is given in Rev. 9.

Rev. 6. 1,*22*,*1*,52,30 written like 1,*23*,52,30.

Rev. 7. 23,26, the difference for 9 years, should be 23,26,*15*.

Translation.

I shall translate only Section 4, the best preserved section; Section 2 is for the greater part restored after the pattern of Section 4, and the remaining sections are so badly damaged that the numbers in them only made sense in isolation.

Section 4.

Rev. 3. From 15,44,31,52,30 to 10,58,23,26,15 (error for 10,*50*,23,26,15) month.

4. [by month] 42,11,15 add and subtract; exactly as when (?) you make up (?) months.

5. . . . whenever it exceeds 15, call it 15. Whenever it is smaller than 11,15,

6. call it 11,15. 42,11,15 per month, 1,22,1,52,30 per year,

7. [2,]20,37,30 per 14 months, 23,26 (error for 23,26,*15*) per 9 years, 4,41,15 per 18
8. [years.] 4,11 completely[17] restores its velocity. From [1]5,44,31,52,30
9. [to 10,]50,23,26,15 4,54,[8,26,15 ...] ....

Commentary.

Section 4 is concerned with a hitherto unattested variant of Column F, the lunar velocity. I shall call it F'. Its parameters are:

$$M = 15;44,31,52,30^{0/d} \qquad P = 13;56,40 = \frac{4,11}{18}$$

$$m = 10;50,23,26,15 \qquad \Pi = 4,11$$

$$\Delta = 4;54,\ 8,26,15 \qquad Z = 18$$

$$d = 0;42,11,15 \qquad \mu = 13;17,27,39,22,30$$

where d is the difference corresponding to one synodic month. The period is precisely that of Column F in System B (the abbreviated and the unabbreviated Column F have the same period), but none of the other parameters is found elsewhere. The monthly difference 0;42,11,15 is larger than the usual difference of 0;42 for F in System A. Incidentally, 42,11,15 is regular.

Several of these parameters are given in the text. M and m are found in Rev. 3, and again in Rev. 8 and 9. The erroneous value for m in Rev. 3 was an obstacle to understanding the text, but fortunately it appears correctly in Rev. 9. I believe that Δ was given in Rev. 9, and I have restored the number accordingly. Π appears in Rev. 8.; one would expect a noun after 4,11, such as months or steps, but no convincing candidate corresponding to šal could be found. The value of d is found in Rev. 4 and 6. Though its period agrees with System B, F' is truncated at precisely the same values as F in System A, viz., 15 and 11;15. In Rev. 5 and 6 the truncation process is clearly described.

This section further gives the change in F' (without sign) corresponding to various time intervals. In Rev. 6 the monthly difference d is repeated, followed by the difference corresponding to one year. *Year* must mean 12 months here, for

$$2\,\Delta - 12d = 1;22,1,52,30$$

which is the value of the text. Next is given the difference for 14 months

$$14d - 2\,\Delta = 0;2,20,37,30$$

[17] The phonetic reading of *šal-meš* was suggested to me by Professor J. J. Finkelstein in a conversation on York Street. It is without precedent in the astronomical literature. My thanks are further due to Professor Finkelstein for kindly checking several of these tablets for me while he was in the British Museum in 1966.

which, but for the initial 2, is found at the beginning of Rev. 7. Since 14 months is only slightly larger than

$$P = 13;56,40$$

this is a good checking parameter for F' (as are the corresponding values for F and Φ).

9 years must here mean 111 synodic months, for

$$8 \cdot 2\,\varDelta - 1,51 \cdot d = 0;23,26,15$$

which is the number written in Rev. 7, though the scribe omitted the final 15.

Finally, the difference for 18 years, i.e., 223 synodic months, is given, viz.,

$$16 \cdot 2\,\varDelta - 3,43 \cdot d = 0;4,41,45.$$

We shall now turn our attention to Section 2 which I have restored after the pattern of Section 4. The phrase in Obv. 6', describing truncation at 15, and the word zi (= velocity) in Obv. 8' identify the subject of this passage as a Column F variant. The difference 4,30 corresponding to 18 years suggests the parameters of the usual, unabbreviated Column F of System A, for which

$$d_{\mathrm{F}} = 0;42$$

$$\varDelta_{\mathrm{F}} = 4;52,49,41,15,$$

and with these values:

$$16 \cdot 2\,\varDelta_{\mathrm{F}} - 3,43 \cdot d_{\mathrm{F}} = 0;4,30$$

(this is the difference corresponding to 17,46,40 for Φ). These parameters further serve to explain the number in Rev. 7' as the difference corresponding to one year (12 months), for

$$2\,\varDelta_{\mathrm{F}} - 12 \cdot d_{\mathrm{F}} = 1;21,39,22,30.$$

The differences corresponding to the other time intervals become:

for 14 months: $\quad 14 \cdot d_{\mathrm{F}} - 2\,\varDelta_{\mathrm{F}} = 0;2,20,37,30$
for 9 years: $\quad 8 \cdot 2\,\varDelta_{\mathrm{F}} - 1,51 \cdot d_{\mathrm{F}} = 0;23,15$

and I have restored the passage accordingly. It should be noted that the difference for 14 months is the same for F and F'.

These parameters depend only on d and $\varDelta$, so they give no information about the extrema. I do not believe that Section 2 deals with the standard

Column F from System A, for in Obv. 5', where the ending of the minimum is expected, we find [. . .]46,52,30 which does not agree with

$$m_F = 11;4,4,41,15$$

nor, for that matter, with

$$M_F = 15,56,54,22,30.$$

Thus I expect that Section 2 presents a variant of F which agrees with the standard function in d, Δ, and therefore P, but not in μ. Even so, this variant is also truncated at the usual values of 15 and 11;15 though, strictly speaking, only the truncation at 15 is described in the preserved part.

All I can safely say about the other sections is that they are concerned with variants of F. In Section 1 we find the characteristic values 15 and 11;15, while I can make no sense of the remaining few and fragmentary numbers. In Section 3 the two characteristic values occur again, as well as zi. The number 13,5,9,22,30 in Rev. 1 is too small to be a mean value of lunar velocity. In Rev. 2 we find 42,11,15 which is the monthly difference of F'.

In Section 5, Rev. 10, occurs the expression *ana tar-ṣa* which is the standard terminology for transforming one function into another[18] (*ana tar-ṣa* (*a*) (*b*) means: opposite (*a*) (put down) (*b*)). I have restored the line to be identical with Obv. I, 18 of ACT No. 200, where the section in question treats of the transformation of Φ into F. It was natural to restore M_F and m_F in Rev. 11., since the preserved numbers agree with the ending of M_F.[18a] These restorations are obviously not very secure, yet it is a possibility that Section 5 is concerned with the standard Column F of System A which, of course, is in phase with Φ.

It is unusual to find variants of functions or models juxtaposed in one text (other examples are ACT Nos. 812 and 813 which present various methods for Jupiter). The purpose of the text is not clear to me, nor is the astronomical justification of truncating variants of F at the same values.

The importance of Text **F** is that it furnishes direct evidence for the truncation of F, giving us the technical terminology for this process. Strictly speaking, it contains the only direct evidence for the truncation below at 11;15, for only rarely is an F-value smaller than 11;15, and the truncation of Φ at 1,58,31,6,40 had to be inferred from the structure of Λ and G.[18b]

[18] cf. ACT II, p. 494.

[18a] There is now a text (B.M. 36722 + 40082) in which it appears that Col. F is truncated at the values 15;25,54,22,30 and 11;25,4,4[1,15]; since the endings agree with those of the extrema, it is not entirely excluded that these effective extrema should be restored here. This text will be published as Text **K** in O. Neugebauer and A. Sachs, *Some Atypical Astronomical Texts II* in the Journal of Cuneiform Studies.

[18b] This is no longer so (see the note added in proof at the end of this paper).

Additions and Comments to the Saros Text

In the present section I shall first present those passages of NEUGEBAUER's Saros text which are affected by the small fragment I joined to it. Unfortunately, the new piece adds but little to our understanding of this difficult text; indeed, in several cases it merely confirms NEUGEBAUER's restorations.

Next, I shall discuss some hitherto obscure parts of this text which can now be understood in light of the knowledge gained from Text **E**. It was particularly the fact that Φ is truncated at 2,13,20 and 1,58,31,6,40 that enabled me to make sense of some of the numbers which previously lacked motivation.

To republish the entire text seemed absurd, so in the following I shall presuppose a familiarity with NEUGEBAUER's publication.

B.M. 37484 (= 80-6-17,1241) joins B.M. 36705 (=(80-6-17,437)+(80-6-17,458)).
Photograph: Plate V.

The small fragment (at most $1\frac{1}{4}$ inches either way) joins the upper right corner of the obverse (= lower right corner of the reverse). Part of the right edge is preserved, and the fragment is near the top edge. It appears, that NEUGEBAUER's Section 1 has a predecessor which I shall call Section 0 to avoid renumbering the sections, and similarly for the line numbers.

Transcription.

Obverse.

Section 0.

−1. [. . . . .] *ù* 2[0 . . . .]

Section 1.

0. [. . . . .] 1,5,4,22,30 DU-*ma*
1. [. . . . . . .2,13],20 *a-na* 1,58,31,6,40 m[u . . .]
2. [. . . . .]E-*ma* 14,48,53,20 mu-m[eš . .] 17,46,40 *taš-pil-tú*
3. *šá* 18 mu-meš *šá* E-ma 13,39,35,36[. . .]6,15 *šu-ú*

Section 2.

4. 17,46,40 a-rá 9,20 DU-*ma* 2,45,55,3[3,20 . . .]. . . 13,46,38,15 me
5. ta *muḫ-ḫi* zi *sin* gal en *muḫ-ḫi* zi *sin* tur . . .[. . .] etc.

Reverse.

Section 15.

32. *a-na tar-ṣi* 2,17,4,48,53,20 2,15,31,6,35,33,20
33. [*a-na t*]*ar-ṣi* 2, . . ,59,15,33,20 4,46,42,57,46,40
34. [*a-na tar-ṣi* 1,57,4]7,57,46,40 5,15,28,23,37,46,40
35. [. . . . .] blank *bi-ri-šú-nu*
36. [. . . . .] ⌊1,22,57,46,40⌋ [. . . .]

Rest destroyed.

Critical Apparatus.

Obv. – 1. 2[0]: *sin* and *šamaš* both possible.

Obv. 1. 40 m[u. . .] written on the edge.

Obv. 4. 13,46,38,15 is preceded by what looks like a high diagonal wedge and two corner wedges. The final 5 and "me" are on the edge.

Rev. 35. From the blank space with a ruling above it Neugebauer concluded that a new section began here. The position of *bi-ri-šú-nu* contradicts this.

Rev. 36. The reading of this number is very tentative. It is, however, unlikely that this is the first line of a new section.

Commentary.

In Obv. 2 and 3 we now have the statement, that *17,46,40 is the difference for 18 years*, which was the basis for our understanding of Φ.

The number 1,5,4,22,30 in Obv. 0, which also occurs farther down in Obv. 9, is, as Neugebauer pointed out, the result of dividing Δ_Φ by 17,46,40. I shall return to it below where it will be called N.

In Obv. 1 we find 1,58,41,6,40, which is the effective minimum of Φ, so I have restored the effective maximum, 2,13,20, just preceding it. The number 14,48,53,20 in Obv. 2 can now be explained as the difference between 2,13,20 and 1,58,31,6,40, i.e., the effective amplitude of Φ; I shall return to it below where it will be called δ_2. I do not understand why these numbers are denominated years,[19] nor can I make sense of the number in Obv. 3 beginning 13,39,35, . . .[20].

In Section 2, Obv. 4 and 5, we now read: *13,46,38,15 days from high velocity of the moon to low velocity of the moon.*

[19] The meaning *line* of mu does not seem to help.

[20] It may be a value for half the sidereal month, but I cannot make it fit the context.

13;46,38,15 days is indeed half the anomalistic month; this value is found in Obv. 6 converted into large hours, as NEUGEBAUER recognised.

In Section 15, Rev. 32, 33, and 34, the new fragment confirms NEUGEBAUER's restoration of the endings of the numbers, and shows further that nothing else was written in these lines. In Rev. 34 we now have the word *bi-ri-šú-nu* (= the distance (arc) between them), which may refer to lunar elongation. Even so, I cannot explain the numbers in this section beyond what NEUGEBAUER already has done. So far the new fragment.

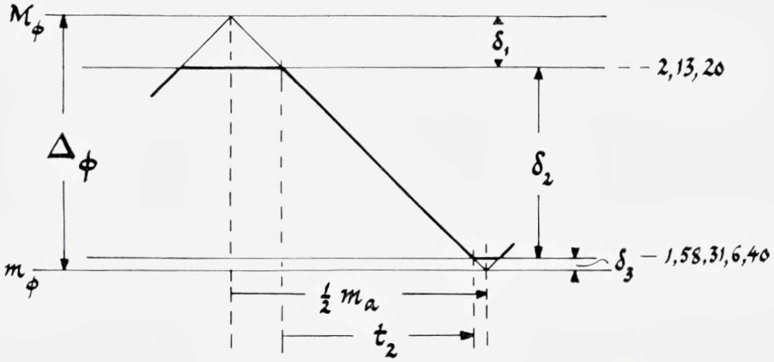

Fig. 6.

The recognition of Φ's effective extrema makes it possible to explain a few more numbers in the Saros text, particularly those in Section 3.

Consider a branch of Φ, subdivided at 2,13,20 and 1,58,31,6,40 (see Figure 6). We introduce

$$\begin{aligned} \delta_1 &= M_\Phi - 2;13,20 &&= 0;3,44,48,53,20^{\mathrm{H}} \\ \delta_2 &= 2;13,20 - 1;58,31,6,40 &&= 0;14,48,53,20^{\mathrm{H}} \\ \delta_3 &= 1;58,31,6,40 - m_\Phi &&= 0;0,43,8,53,20^{\mathrm{H}} \end{aligned}$$

so

$$\delta_1 + \delta_2 + \delta_3 = \Delta_\Phi = 0;19,16,51,6,40.$$

If we now proceed along the branch of Φ in steps of length 0;0,17,46,40, as we did in Text **E**, we find, dividing the δ's by 0;0,17,46,40, that the corresponding number of steps are:

from M_Φ to 2;13,20 : $n_1 = 12;38,45$ steps
from 2;13,20 to 1;58,31,6,40 : $n_2 = 50$ steps
from 1;58,31,6,40 to m_Φ : $n_3 = 2;25,37,30$ steps

and

from M_Φ to m_Φ : $N = n_1 + n_2 + n_3 = 1,5;4,22,30$ steps.

The number N occurs in Obv. 9, and now in Obv. 0, and was correctly explained by NEUGEBAUER.

Section 3 contains n_1, n_2, and n_3 and instructs us to multiply them by 17,46,40. The results, which are not given, are δ_1, δ_2, and δ_3.[21]

In Obv. 9, 10, and 11, the length of half the anomalistic month (in large hours) is divided by N, yielding

$$t = \frac{1,22;39,49,30^{\mathrm{H}}}{1,5;4,22,30} = 1;16,13,10,11,24,36^{\mathrm{H}},$$

as NEUGEBAUER recognised though, as he points out, the division is not executed quite correctly. If we now consider again the branch of Φ, i.e., the true Φ-function whose period is the anomalistic month, then t is the time it takes to move one step of length 0;0,17,46,40 along this branch.

The time t_2 it takes to travel from 2;13,20 to 1;58,31,6,40 is thus:

$$t_2 = n_2 \cdot t = 50 \cdot t = 1,3;30,58,29,30,30^{\mathrm{H}}$$

which is the number preserved in Rev. 3 (what NEUGEBAUER read as "$\frac{1}{2}$" is "*ina* 1" written closely so the horizontal wedge of *ina* intersects the vertical 1, and where he read "?48" one should read 58).

[21] NEUGEBAUER carries out these multiplications and gives the results on page 18 of the Saros paper (there are, unfortunately, two errors in the printing: 3,44,*50*,53,20 should be 3,44,*48*, 53,20 ($= \delta_1$) and *48*,8,53,20 should be *43*,8,53,20 ($= \delta_3$)).

It is remarkable that several of the parameters of Φ are simple multiples of 17,46,40. Thus, e.g.,

$$2;13,20 = 7,30 \cdot 0;0,17,46,40$$
$$1;58,31,6,40 = 6,40 \cdot 0;0,17,46,40.$$

For the constants c and l mentioned above on page 11 we also have

$$c = 10,0 \cdot 0;0,17,46,40$$
$$l = 6,15 \cdot 0;0,17,46,40$$

though l, of course, is measured in degrees.

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Added in Proof

Just after the manuscript of this paper went to the printer I came across two texts in the British Museum that contain additional information about F and Λ.

The first text (BM 36961 (= 80–6–17,702)) is a fragment containing remnants of three columns. What is of interest here is that the second is the abbreviated Column F of System A whose parameters are:

$$M = 15;57 \qquad d = 0;42$$
$$m = 11;4$$

and it is truncated at 15 and 11;15. Since the first line has 14;40 ↓, the effective minimum 11;15 actually appears in line 6. The effective maximum 15 occurs thrice (in lines 12–14). It is worthy of note that on the upper edge is visible $^{\text{I}}$] *Se-lu-ku,* so the practice of truncating F was certainly known in Seleucid times.

The second text (BM 40094 (= 81–2–1,59)) is of considerable interest. It is a fragment of an ephemeris for consecutive new moons; though the date column is destroyed, I can say with confidence, on the basis of internal evidence, that the text covers the three years from S.E.–8, XII to S.E.–5, XII. Thus it antedates the oldest hitherto known lunar ephemeris (ACT No. 1) by 132 years. It can further be shown, again on indirect, but secure evidence, that solar longitudes not only were computed according to the well-known System A scheme, but also that they are continuable o the ACT texts, and similarly for Column Φ. What is of particular concern here is that this text contains a Column Λ (month by month)--this is the first time Λ has been encountered in an ephemeris–and following this a column (I call it Column Y) giving corrections for solar anomaly to Λ. Y is related to J (the analogous correction of G) by the rule that the monthly difference in Y is the 12-monthly difference in J. Further, Column Y, as Column J, is 0 on the fast arc. It is presently not out of the question that the connexion between the initial values (or mean values) of G and Λ is to be found here, and that it rests on the decision that both J and Y be 0 on the arc of high monthly progress of the sun.

Indleveret til Selskabet den 13. september 1967.
Færdig fra trykkeriet den 12. februar 1968.

PLATES

Text A

Text B, Obv.

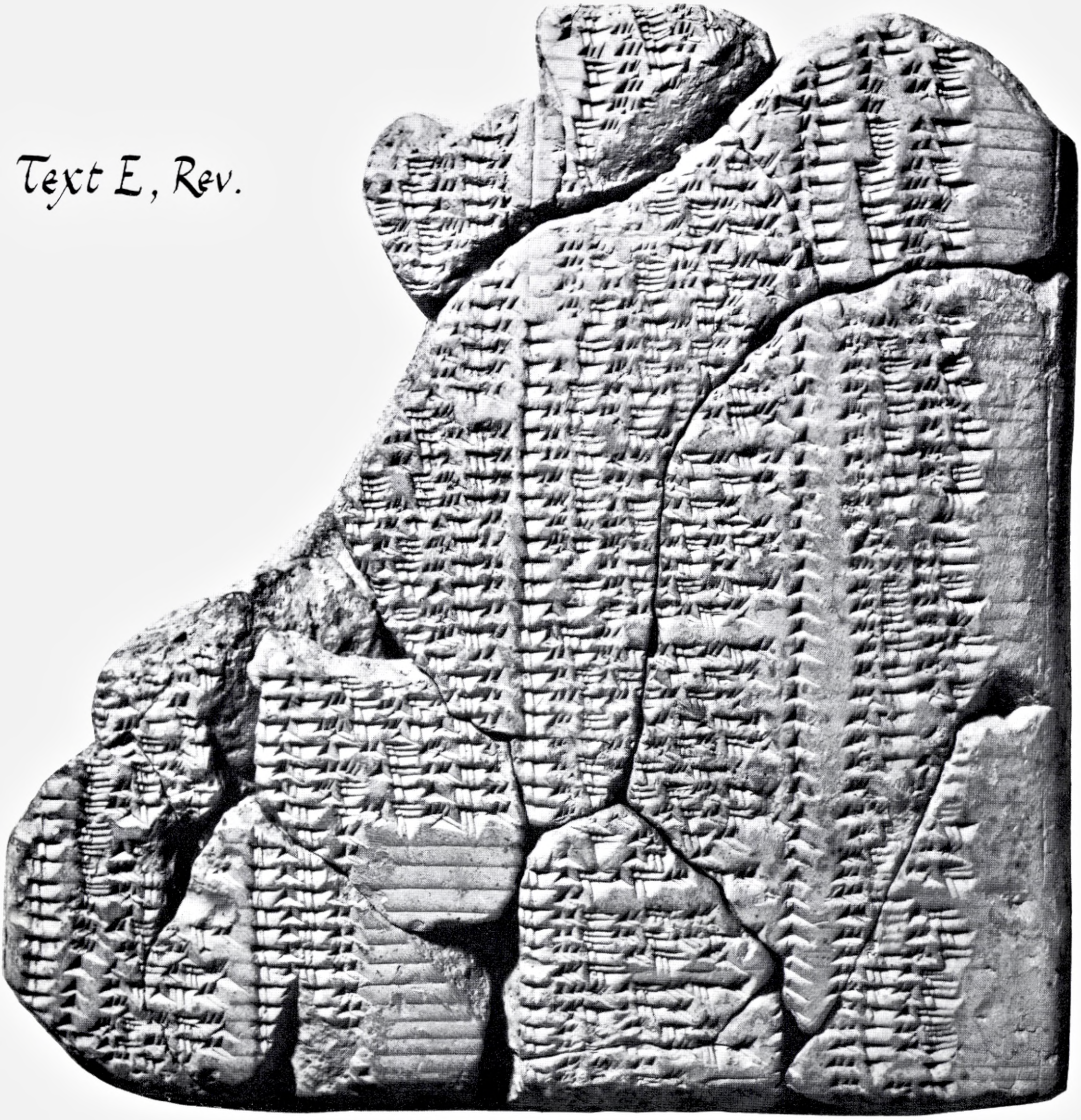
Text C

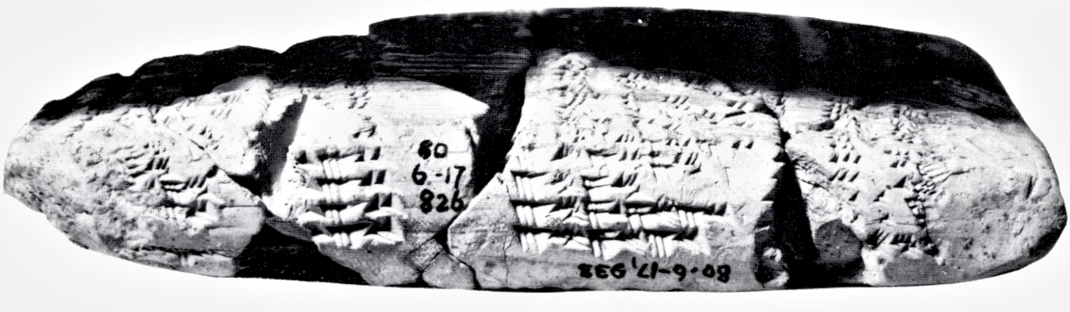
Rev.

Text D

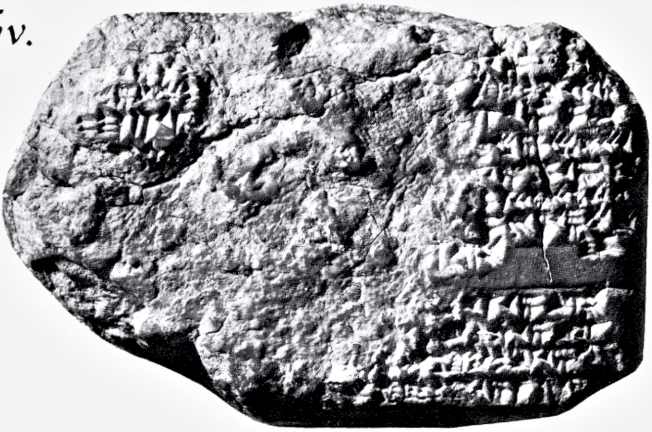
Text E, Obv.

Text E, Rev.

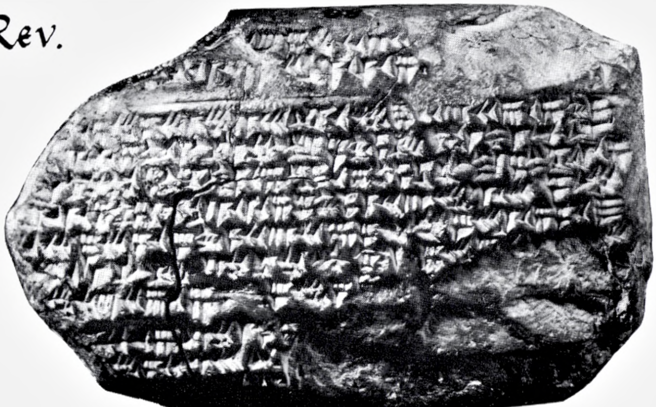




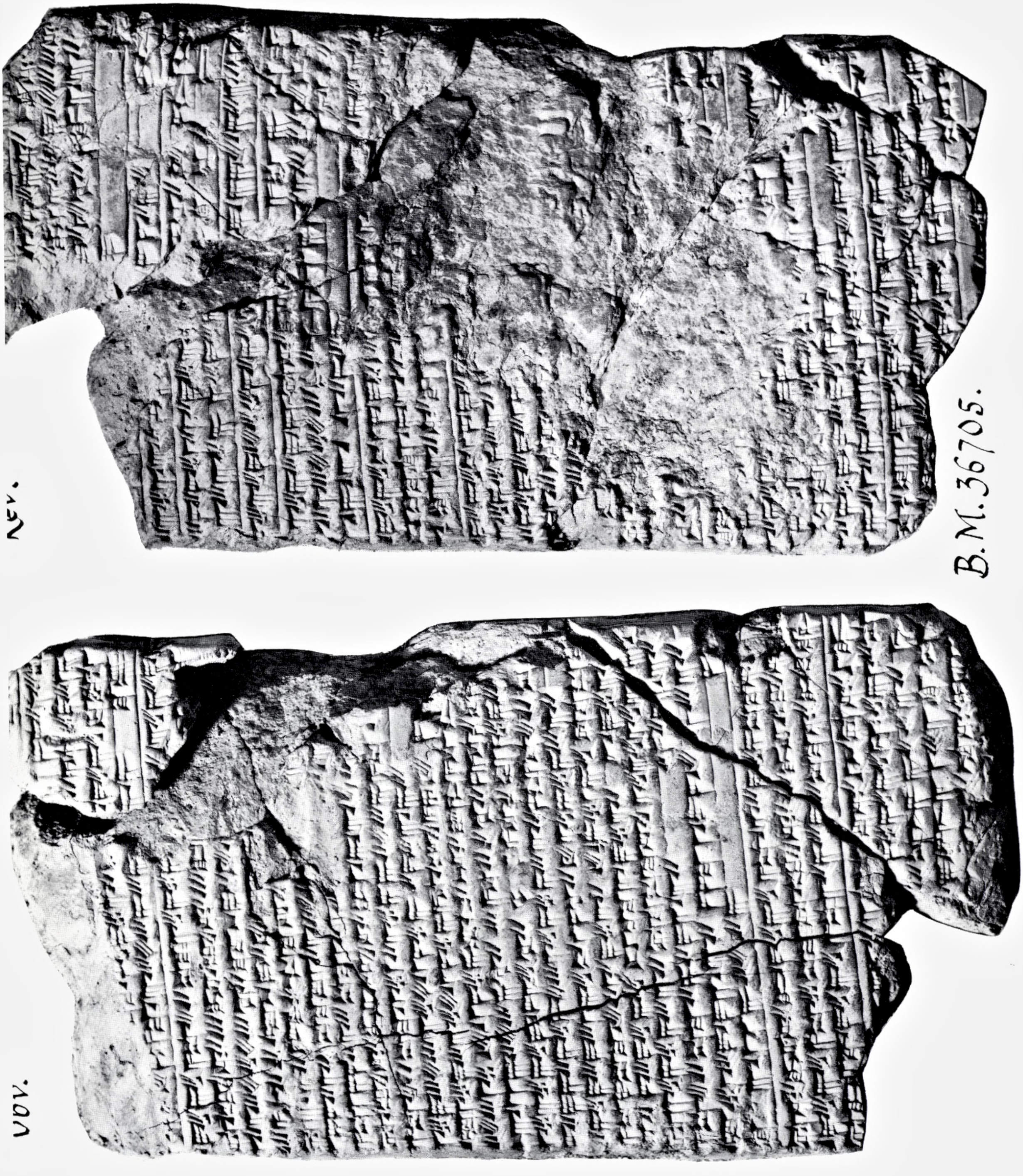
Obv.



Rev.



Text F



B.M. 36705.